Journal of the Institution of Engineers, Bangladesh
Vol. 16, No. 4 Oct. 1988

Analytical Study on Minimization of Edge Beam in Long Cylindrical Shells

Salek Muhammad Seraj* and Alamgir Habib*

Abstract :

The results of an analytical investigation of long cylindrical shells with edge beam have been presented in this paper. A computer programme using the Schorer Theory has been developed to determine the stress resultants and principal stresses in the shell at different locations. The programme also finds out stresses and forces in the edge beams and calculates the amount of steel required. The programme finally has a check for statics. This programme has been used to find out the minimum allowable size of the edge beam conforming to the allowable stresses in the materials and the relevant ACI building code provisions. The results obtained have been presented in graphical form. An outline of the computer programme is also given in the paper.

Introduction :

Reinforced concrete thin shells are being increasingly used in Bangladesh. Maximum structural advantage out of a minimum amount of material is achieved by using shell froms. In designing relatively long cylindrical shells, edge beams are needed to reduce the forces and moments in the shell (Fig. 1). The dimensions of the edge beams remain to be chosen by the designer who faces the real trouble in doing so. Large edge beams often become aesthetically and functionally unacceptable. On the contrary, smaller edge beams may not fulfil structural demands. So the designer has to give a series. of trials to know the minimum edge beam dimension. But the rigorous analysis of thin shells are so extra ordinarily complex that the designer often chooses a somewhat larger edge beam to minimize design time and hence save design expenditure. Our present study aims at providing some guidelines to the designer for selection of the minimum dimension of the edge beam.

Theory:

In a cylindrical shell under bending, there are, in general, 10 stress resultants to be detdrmined (Fig 2). The Schorer Theroy, which is applicable to long shells only, has been used in this study. The Schorer Theory is based on the following assumptions :

i) Poisson's ratio may be ignored, i.e $\gamma = 0$

**Department of Civil Engineering, BUET.*

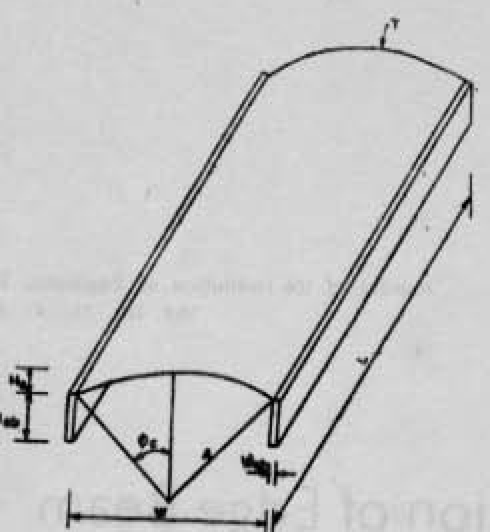

Fig. 1 Long Cylindrical Shell With Edge Beams.

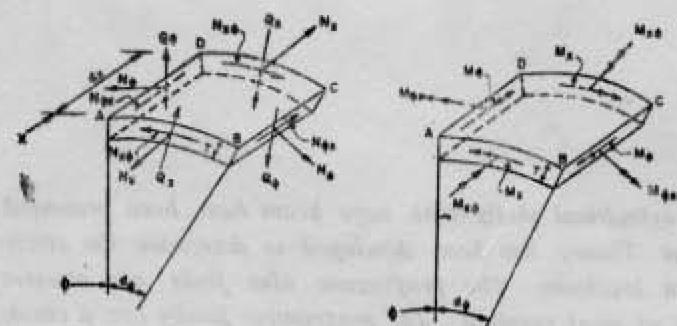

Fig. 2 Differential Shell Element Showing Stress Condition.

ii) The longitudinal bending resultant M_x is small in comparison with the transverse bending resultant $M\phi$ and thus M_x may be ignored, i.e. $M_x = 0$.

iii) The stress resultants $M_x\phi$ and $M\phi_x$ are small in comparison with $M\phi$ and may thus be ignored, i.e. $M_x\phi = M\phi_x = 0$.

iv) The dominant strain is that in the longitudinal direction (x direction), namely ϵ_x, and that by comparison $\epsilon\phi$ and δ_{xy} may be ignored, hence $\epsilon\phi = \chi_{xy} = 0$.

The detail expressions for stress resultants and displacements of the Schorer Theory are available in any standard text [6].

Scope of the Study :

Schorer has fixed a limit span/radius $(L/A) \geq \pi$ for his theory to be valid. In this study Schorer Theory has been applied varying L/A from 2.5 to 5.0. The values of thickness (T), Semicentral angle (ϕ_c) and width of edge beam (W_{eb}) are 0.25 ft., 35 degrees and 0.833 ft respectively. The live load chosen is 12.5 psf of shell surface.

Minimization Criteria :

In this study the minimization criteria for the selection of edge beam dimensions are based on the requirements of ACI Building Code [1]. Moreover, the dimensions are so chosen that the stresses in the structural members are kept within the allowable limits. The criteria used in the search of minimum edge beam size are listed below.

i) Shell reinforcement : Area of shell reinforcement in square inches per foot of shell width shall not exceed $7.2\ h\ f'_c/f_y$ nor $29000\ h/f_y$.

ii) Allowable stress : Concrete in compression 1350 psi. Steel in tension-20000 psi.

A separate set of curves has been prepared with the additional criterion that the maximum deflection of the edge beam is limited to L/360.

Computer Programme :

A computer programme, based on the Schorer Theory has been developed in FORTRAN language. The schematic flow-diagram is out lined in Fig. 3.

Analytical Results :

The computer programme is used to find out the minimum value of height of edge beam (H_{eb}) upto 0.01 ft. accuracy. Width of shell (W) has been varied from 22.5 ft. to 50.0 ft. in intervals of 2.5 ft. The H_{eb} vs. L/A for different values of W and height of the shell top from the top of the edge beam $(H_s)/H_{eb}$ vs. L/A for different values of W are presented in Fig. 4,5,6 and 7.

Numerical Example :

The size of the edge beams of design examples in some of the standard text books have been compared in the following articles with the corresponding sizes obtained from the curves presented in this paper.

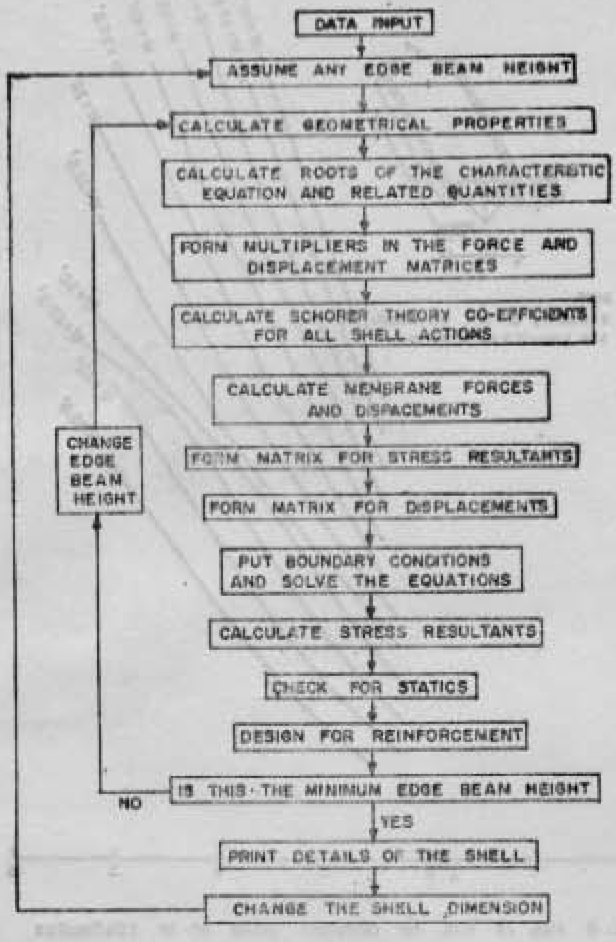

Fig. 3 Schematic Flow Diagram of The Shell Programme for Determining The Minimum Size of The Edge Beam.

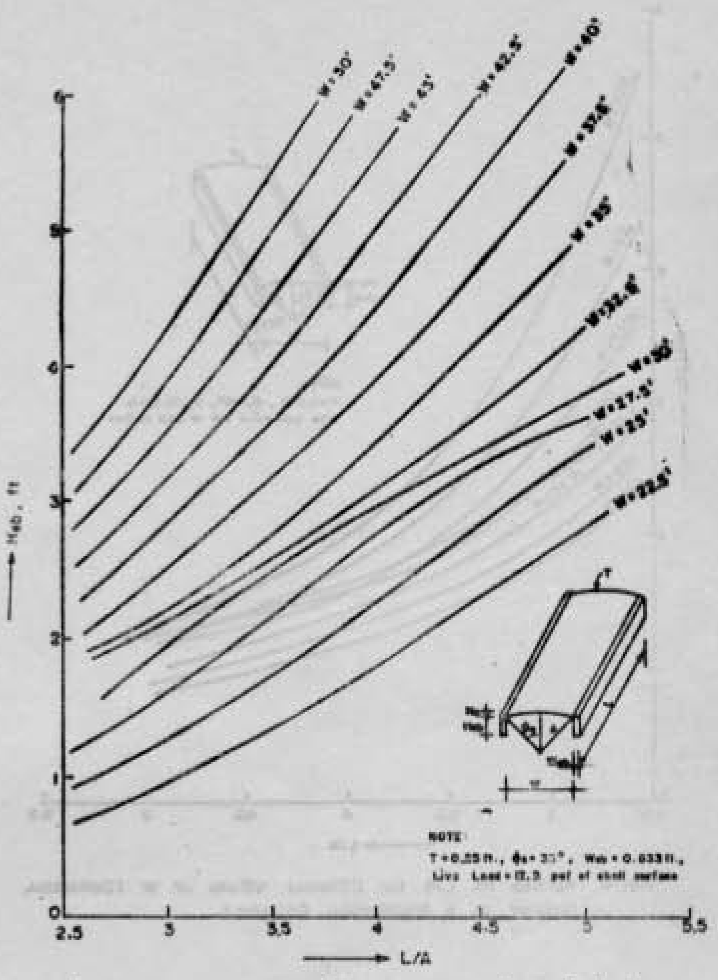

Fig. 4 H_{eb} vs. L/A for Different Values of W (Deflection ignored as a Minimization Criterion).

Example 1 :

In the Design Example 8-2 of Ref. 6, L=83.25 ft. A=25 ft., T=0.25 ft. $\phi_c=35°$, $W_{eb}=0.75$ ft. and live load = 12.5 psf. The height of the edge beam for this shell was selected to be 5.0 ft.

For this problem L/A = 3.33 and W = 28.67 ft. Using Fig. 4 of this paper the minimum height of the edge beam (H_{eb}) comes out to be 2.15 ft., i.e. 43% of the height of the edge beam used in the text book.

Example 2 :

In the Art. 6.1 of Ref. 5, L = 120 ft., A=30 ft., T = 0.25 ft., $\phi_c = 40°$, $W_b = 0.833$ ft. and live load = 12.5 psf. The height of the edge beam for this shell was selected to be 5.0 ft.

For this problem L/A = 4.0, W = 38.57 ft. Using Fig. 4 of this paper H_{eb} comes out to be 4.2 ft. which is less than the edge beam used in the text book.

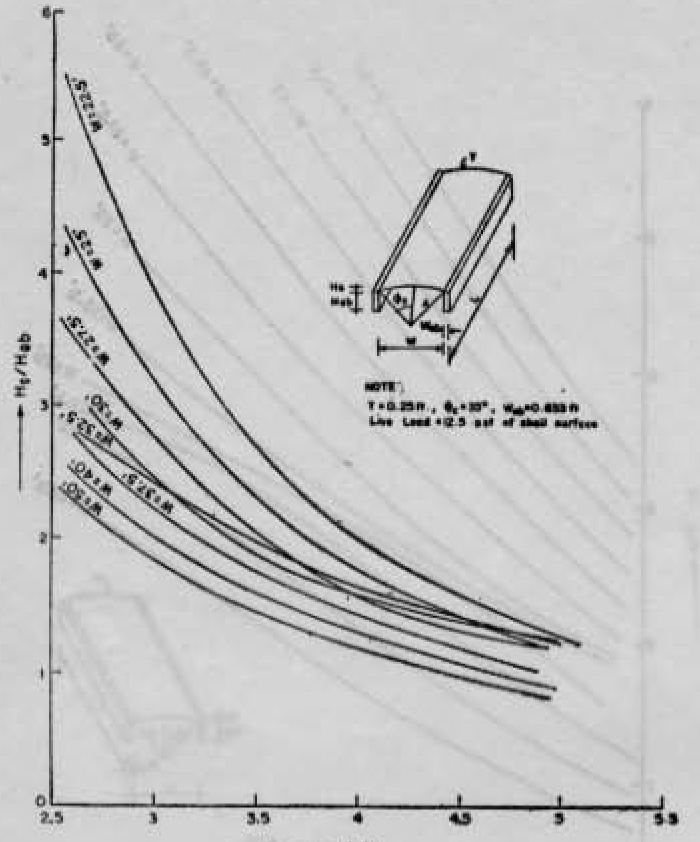

Fig. 5 Hs/Heb vs. L/A for Different Values of W (Deflection Ignored as a Minimization Criterion)

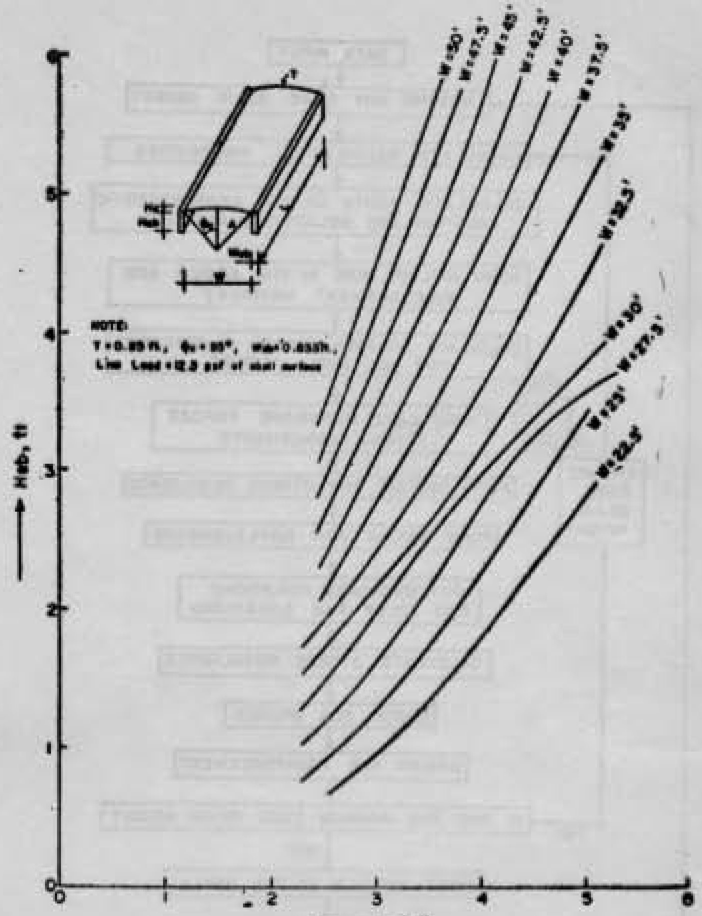

Fig. 6 Heb vs L/A for Different Values of W (Deflection Considered as a Minimization Criterion).

Example 3 :

In the Example 3 of Ref. 2, L = 111 ft., A = 33.33 ft., T = 0.333 ft., $\phi_e = 30^o$, $W_{eb} = 0.666$ ft. and show load = 25 psf. The height of the edge beam for this shell was selected to be 5.0 ft.

For this problem L/A = 3.33 and W = 33.33 ft. Using Fig. 4 of this paper H_{eb} comes out to be 2.5 ft. which is much less than the edge beam used in the text book.

Conclusion :

The curves given in this paper are based on the Schorer Theory. However, these curves may also be used as a guide for selecting the depth of edge beams when shells are designed by using any other theory. It has been noticed from the curve and corresponding computer output that for the long shells with relatively larger shell width, permissible deflection, if used as a criterion, governs the minimization process. For shells with smaller width, maximum permissible reinforcement prescribed by the code is the governing factor. Concrete stress in compression has never acted as a governing limit,

Acknowledgement :

The computer programme used in this study was developed on the IBM 4331-K02 computer system at

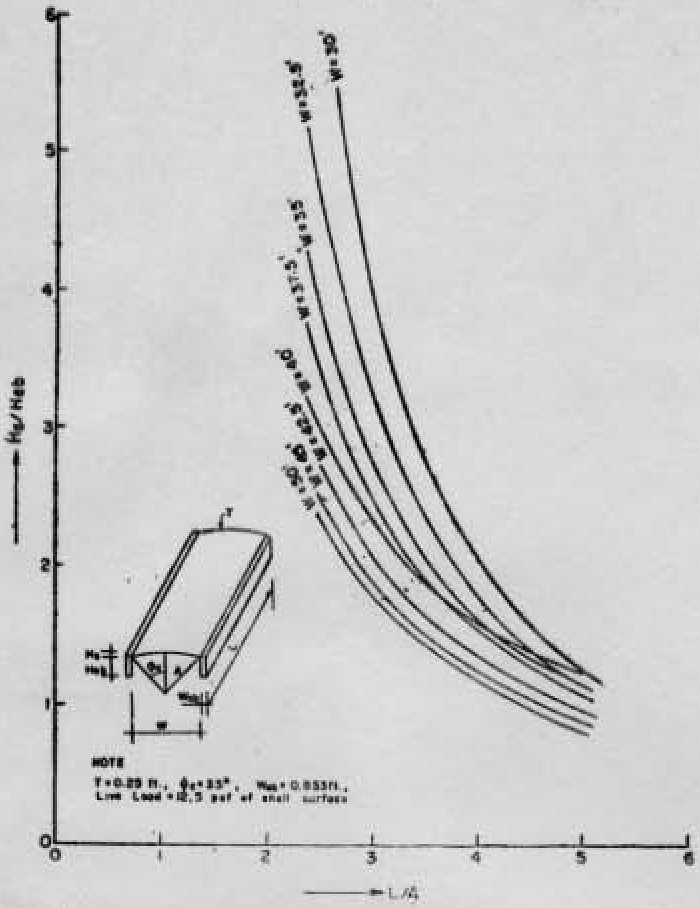

Fig. 7 H_s/H_{eb} vs. L/A for Different Values of W (Deflection Considered as a Minimization Criterion)

BUET Computer Centre. The co-operation and assistance of the Computer Centre personnel are gratefully acknowledged. Thanks are also due to Mr. Md. Shahiduddin for his help during preparation of the diagrams of this paper.

Notation :

- L = span of the shell, ft.
- W = Width of the shell, ft
- ϕ_e = semicentral angle
- W_{eb} = Width of the edge beam, ft.
- $\overline{T}$ = thickness of the shell, ft.
- A = radius of the circular directrix, ft $= W/(2 \sin\phi_e)$
- H_s = height of the shell top from the top of the edge beam, ft
  $= A - A \cos \phi_e$
- H_{eb} = height of the edge beam, ft
- h = overall thickness of the member, in
- f'_c = specified ultimate compressive strength of concrete, psi
- f_y = specified yield strength of non-prestressed reinforcement, psi
  = 40000 psi.

References :

1. ACI, "Building Code Requirements for Reinforced Concrete", ACI 318-77.
2. ASCE, "Manuals of Engineering Practice No. 31-Design of Cylindrical Concrete Shell Roofs", 1952.
3. Billington, D.P., "Thin Shell Concrete Structures", McGraw Hill Book Company, 1965.
4. Gibson, J. E., "Linear Elastic Theory of Thin Shells," Pergamon Press, 1965.
5. Gibson, J.E., "The Design of Shell Roofs", E & F. N. Spon Ltd., London, 1968.
6. Ramaswamy, G.S., "Design and Construction of Concrete Shell Roofs," McGraw Hill Book Company, 1968.