APPLICATION AND ASSESSMENT OF A PROPOSED RESPONSE SPECTRA

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ABSTRACT - In one of the companion papers (Ansary, et al., 1998), efforts have been made to formulate response spectra based on simulated earthquake ground motion. In this study, performance of the proposed spectra have been tested against Uniform Building Code spectra of 1994 by analysing various moderately high moment resisting framed structures. In order to evaluate the performance of the developed spectra, on building structures subjected to seismic forces, a typical beam column frame structure has been selected. Base shear of the building which has been calculated for different time periods using a general purpose finite element package STRAND6, has been selected as the criterion for comparison. Natural frequencies of buildings have been calculated considering short direction, long direction and the building as a whole. The dynamic analysis procedure uses a response spectrum representation of the seismic input motions. Since maximum modal responses will not occur at the same time during the earthquake ground motion, Complete Quadratic Combination method with 5 percent of critical damping has been considered for modal combination to estimate the maximum composite response of the structure. It has been observed from this study that, for directional analysis in the three-dimensional continuum, any mode shapes transverse to the direction and torsional modal shapes have no participation in the final result of the structure. It has also been observed that ordinates of the proposed response spectra are greater than UBC spectra at lower and higher time periods. Here, higher spectral values at lower time periods have been proposed. Faster attenuation has been modified and an attenuation rate has been suggested which is inversely proportional to time period.

NOTATION

I	Importance factor
Z	Zone factor
R_w	Numerical coefficient depend on basic structural system
N	Significant number of modes
S_{ak}	Spectral acceleration in the k^{th} direction
P_k	Building's modal participation factor
$^n(\phi_{ik})$	Mode shape amplitude for n^{th} mode, i^{th} nodal point and k^{th} direction
M_{ik}	Component of mass for i^{th} node point and k^{th} direction
^nM	Modal mass for n^{th} mode
NP	Total number of node points
NK	Total number of directions of motion at a node point
OM_k	Overturning moment due to lateral forces in k^{th} direction
F_{ik}	Lateral force at i^{th} node point in k^{th} direction
V_k	Base shear force in k^{th} direction
h_i	Vertical distance from i^{th} node point to base of building
u_k	Structure's maximum composite response
ρ_{ij}	Cross-modal coefficient
r	Ratio of natural period of j^{th} mode, T_j, to natural period of i^{th} mode, T_i
T_i	Natural period of i^{th} mode
T_j	Natural period of j^{th} mode
$[K]$	Banded stiffness matrix
$\{x\}$	Eigenvector
$[M]$	Mass matrix
χ	Eigenvalues

INTRODUCTION

Modal analysis requires that the design spectrum be specified. Many building codes stipulate either a design acceleration spectrum or a base shear coefficient as a function of natural period. These coefficients are essentially ordinates of acceleration spectra divided by the acceleration of gravity; the relationship holds exactly in single-degree systems (Newmark and Rosenblueth, 1971). Several widely differing methods (Newmark and Hall, 1974) exist for specifying the design earthquake. Some of these methods are equivalent-static loading in codes, response spectra, records of real earthquakes and theoretical simulation. Ansary, et al. (1998) has described the procedure to develop modified response spectra from simulated earthquake data. In this study, these developed spectra have been compared with the corresponding spectra proposed by Uniform Building Code (UBC, 1994). Efforts have also been made to propose a design response spectra for dynamic structural analysis. A coefficient to be used in equivalent static load method to cater for various soil types and varying building time periods have been derived.

Efforts have been geared towards arriving at an all encompassing response spectra which may either be readily used in dynamic analysis or may be conveniently adopted in static analysis by deriving a suitable numerical co-efficient from it. During the course of the study a wide range of synthetic earthquake data have been generated. The design aids have been compared with Uniform Building Code (1994) provisions by analysing various moderately high tall buildings.

BUILDING ANALYSIS USING PROPOSED AND UBC SPECTRA

In order to evaluate the performance of the developed spectra, using simulated earthquakes, on building structures subjected to seismic forces, a typical beam column frame structure has been selected. The typical floor plan of the building that was selected for this study is shown in Figure 1. The developed spectra have been compared with the existing Uniform Building Code (1994) spectra. Base shear of the building, which has been calculated for different time periods using the package STRAND6 (1996), has been selected as the criterion for comparison. A brief description of the response spectrum analysis method, which has been used here, has also been included. For the purpose of spectral analysis, base acceleration has been applied to the direction parallel to the short planar dimension of the building. Natural frequencies of buildings have been calculated considering short direction, long direction and the building as a whole. When short or long direction analyses were performed, degrees of freedom of other directions were kept restrained. This has been done to minimise the computer running time. Short direction of the building has been used for further analyses. The importance factor (I) and the zone factor (Z) have been taken as 1.0 and 4.0, respectively, for the purpose of response spectrum analysis. A value of $R_w = 12$ has been used in the analysis. Square-root-sum-of-the-squares (SRSS) and Complete Quadratic Combination (CQC) methods have been used for combination of modes to get the maximum spectral ordinate. In the present study CQC method with 5 percent of critical damping has been considered for modal combination. Two different spectra have been chosen for the purpose of response spectrum analysis. The chosen spectra were (a) UBC (1994) spectra, and (b) Spectra developed using simulated earthquakes (Ansary, et al., 1998).

Description of model building

A typical floor plan and elevation of the building adopted for this study has been shown in the Figure 1. The columns, beams and slabs have constant cross section throughout the height of the building. Although the uniformity and symmetry used in this example have been adopted primarily for simplicity, these are generally considered to be sound engineering design concept, which should be utilised wherever practicable for seismic design. Although the member dimensions used in the example are within the practical range, the structure itself is a hypothetical one and has been chosen mainly for research purposes. Superimposed dead load has been taken equal to 42.5 psf (2.06 kN). Weight and ultimate crushing strength of concrete have been take as 150 pcf (23.56 kN/m^3) and 4000 psi (27576 kN/m^2),respectively. The member dimensions selected for this design were, beams 24 in. (60.96 cm) wide x 26 in. (66.04 cm) deep in transverse direction, beams 24 in. (60.96 cm) wide x 20 in. (50.8 cm) deep in longitudinal direction, columns 24 in. (60.96 cm) by 24 in. (60.96 cm) and slabs 7 in. (17.78 cm) thick.

On the basis of the given data and the dimensions of the building, the weight of each floors comes out to be approximately 6650 kN. The weight of a typical floor includes that of all the elements located between two imaginary parallel planes passing through the mid height of the columns above and below the floor considered. It should be noted that the building has the same lateral load resisting system in both the principal directions. Thus, the lateral seismic forces will be the same in both the longitudinal and the transverse directions of the building. However, since the building is rectangular rather than square in plan, the lateral shears produced by torsion will most likely not be equal in the two directions.

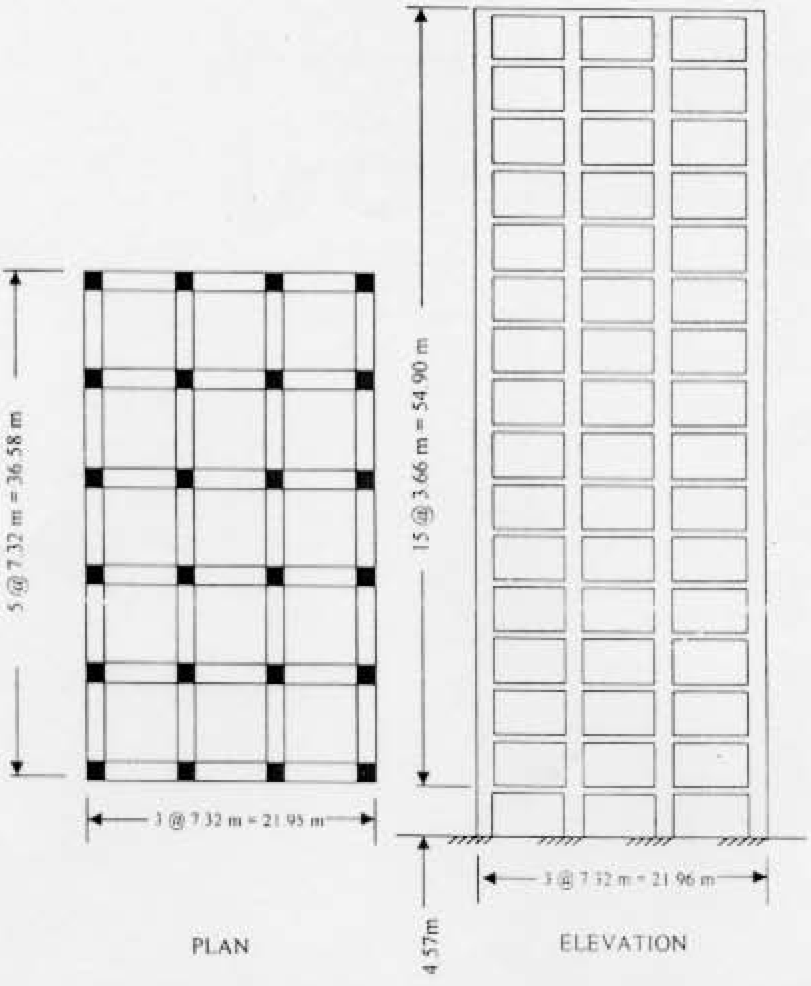


Figure 1. Typical plan and elevation of moment resisting concrete building

Response spectrum analysis

The dynamic analysis procedure described here uses a response spectrum representation of the seismic input motions. The procedure is applicable to linear elastic building models developed in accordance with the requirements of Uniform Building Code (1994). It consists of the following steps (Clough and Penzien, 1975, Biggs, 1954):

a) Principles of mechanics have been used to compute the natural period and mode shape for the first N normal modes of the building model, where N is the significant number of modes.

b) The response spectrum at the natural period of the n^{th} normal mode, $^{n}(T)$, have been entered to obtain the corresponding spectral acceleration in the k^{th} direction, $^{n}(S_{ak})$. Here k may be the x or y horizontal direction or the z vertical direction. Table 1 lists the spectral ordinates for 10 and 16 storey buildings.

c) This spectral acceleration has been used together with the mode shapes and the model's consistant/lumped mass values to compute the building's modal participation factor $^{n}(P_k)$ for the n^{th} mode and the k^{th} direction corresponding to that of $^{n}(S_{ak})$ as shown in Eqn. 1.

$$^{n}(P_k) = \sum_{i=1}^{NP} \frac{^{n}(\phi_{ik})(M_{ik})}{^{n}(M)} \tag{1}$$

Where:

$_n(\phi_{ik})$ = Mode shape amplitude for the n^{th} mode, i^{th} nodal point and the k^{th} direction.
M_{ik} = Component of mass for the i^{th} node point and the k^{th} direction.
^nM = Modal mass for n^{th} mode

$$= \sum_{i=1}^{NP}\sum_{k=1}^{NK} {}^n(\phi_{ik})^2 (M_{ik}) \tag{2}$$

NP = Total number of node points.
NK = Total number of directions of motion at a node point.

The minimum of 90 percent of the participating mass, required in different seismic codes, represent an approximate basis for providing an acceptable degree of accuracy in the computed value of the dynamic base shear; and hence, the representation of all significant modes. Participation factor for the 10 and 16 storey buildings are listed in Table 2.

Table 1. Natural Frequency and Corresponding Spectral Value for 10 and 16 Storey Building. (Soil Type II, UBC, 1994, Short Direction.)

Mode	10 storey building		16 storey building		Damping Ratio
	Frequency	Spectral Value	Frequency	Spectral Value	
1	0.657191	1.13	0.406120	0.82	0.05
2	2.007950	2.39	1.234592	1.73	0.05
3	3.500128	2.50	2.153795	2.5	0.05
4	5.103582	2.50	3.081435	2.5	0.05
5	6.445510	2.50	4.070213	2.5	0.05
6	6.871075	2.50	4.236972	2.5	0.05

Table 2. Seismic Mass Participation Factors for 10 and 16 Storey Building. (Soil Type II, UBC, 1994, Short Direction and Whole Building)

Mode	10 storey building		16 storey building	
	Short direction	Whole building	Short direction	Whole building
1	84.43	0.00	82.08	0.00
2	9.73	84.44	10.60	82.08
3	3.05	0.00	3.30	0.00
4	1.37	0.00	1.59	0.00
5	0.00	9.73	0.88	10.60
6	0.69	0.00	0.00	0.00
7	-	0.00	-	0.00
8	-	3.05	-	3.30
9	-	0.00	-	0.00
Total	99.28	97.22	98.48	95.98

d) The above parameters have been used to compute the peak value of any building response quantity when the building is vibrating in its n^{th} normal mode. For example peak acceleration at i^{th} node point and in k^{th} direction is given in Eqn. 3.

$$^{n}(\ddot{u}_{ik})={}^{n}(P_{k})\times^{n}(\phi_{ik})\times^{n}(S_{ak}) \tag{3}$$

Also, for response in any other direction j is given in Eqn. 4.

$$^{n}(\ddot{u}_{ij})={}^{n}(P_{k})\times^{n}(\phi_{ij})\times^{n}(S_{aj}) \tag{4}$$

Lateral force at i^{th} node point in k^{th} direction has been calculated using Eqn. 5.

$$^{n}(F_{ik})=(M_{ik})\times^{n}(\phi_{ik})\times^{n}(P_{k})\times^{n}(S_{ak}) \tag{5}$$

Base shear force in k^{th} direction has been calculated using Eqn. 6.

$$^{n}(V_{k})={}^{n}(P_{k})^{2}\times^{n}(M)\times^{n}(S_{ak}) \tag{6}$$

Overturning moment due to lateral forces in the k^{th} direction has been calculated using Eqn. 7.

$$^{n}(OM_{k})=\sum_{i=1}^{NP}M_{ik}\times^{n}(\phi_{ik})\times^{n}(P_{k})\times^{n}(S_{ak})(h_{i}) \tag{7}$$

Where h_i is the vertical distance form the i^{th} node point to the base of the building.

e) Steps b through d have been repeated for each of the N normal modes. Then, the resulting peak modal response quantities have been combined for each mode using the procedure described in the next section, in order to estimate the composite peak response value.

Combining modes

The response spectrum analysis procedure provides the maximum responses of the structure when it is vibrating in each of its significant normal nodes. However, because these maximum modal responses will not occur at the same time during the earthquake ground motion, it is necessary to use approximate procedures to estimate the maximum composite response of the structure. Such procedures are typically based on an appropriate combination of the maximum individual modal responses, and should account for possible interaction between any closely spaced modal responses that may exist.

A simple and accurate modal combination approach that satisfies this requirement is the Complete Quadratic Combination (CQC) method (Wilson et al., 1981, Der Kiureghian, 1981 Der Kiureghian, 1980 and Wilson and Bolton, 1982). This approach is based on random vibration concepts and assumes that:

The duration of the earthquake shaking is long when compared to the fundamental period of the structure; and

The design response spectrum exhibits slowly varying amplitudes over a wide range of periods that include the dominant modes of the structure. On this basis, the CQC method leads to the following expression for the structure's maximum composite response, u_k, at its k_{th} degree of freedom:

$$u_{k}=\left[\sum_{i=1}^{N}\sum_{j=1}^{N}u_{ki}\rho_{ij}u_{kj}\right]^{1/2} \tag{8}$$

Where u_{ki} and u_{kj} correspond to the structure's maximum modal response in its k^{th} degree of freedom when it is vibrating it in its i^{th} and j^{th} mode, respectively, and ρ_{ij} is the cross-modal coefficient. It is to be noted that here, u_k, u_{ki}, u_{kj} are general symbols and may correspond to total acceleration, relative (to base) displacement, inter storey drift, base shear, overturning moment, or any other structural response quantity. Furthermore, when computing u_k in accordance with the above expression, the signs of u_{ki} and u_{kj} should be preserved.

The cross-modal coefficient ρ_{ij} as denoted above is dependent on the damping ratios and the natural periods of the i^{th} and j^{th} mode. When the modes have identical damping ratios ξ, then ρ_{ij} is expressed as :

$$\rho_{ij} = \frac{8\xi^2(1+r)r^{3/2}}{(1-r^2)^2+4\xi^2 r(1+r)^2} = \rho_{ji} \tag{9}$$

where r is the ratio of the natural period of the j^{th} mode, T_j, to the natural period of the i^{th} mode, T_i (that is, $r = T_j/T_i$).

From Eqn. 9, it can be shown that: a) $\rho_{ij} = 1$ when $r = 1$; and b) ρ_{ij} decreases with decreasing r in a manner that is dependant on the modal damping ratio ξ. Furthermore, when the modal periods are well spaced such that:

$$r = \frac{T_j}{T_i} \le \frac{0.1}{0.1+\xi}(T_i > T_j) \tag{10}$$

then: $\rho_{ij} \cong 1$ (i = j) and the CQC expression for computing the maximum composite response given in Eqn. 8 becomes:

$$u_k = \left[\sum_{i=1}^{N} u_{ki}^2\right]^{1/2} \tag{11}$$

That corresponds to the square-root-sum-of-the-squares (SRSS) mode combination approach. This shows that the SRSS approach is a special case of the more general CQC method, and can be applied when the modal periods are sufficiently well spaced in accordance with Eqn. 10. Furthermore, the quantities ρ_{ij} for $i \neq j$ can be visualized as corrections to the SRSS approach in order to incorporate effects of coupling between closely spaced modes. As shown in Figure 2, these coupling effects become more important as the modal damping ratio increases. Also, these effects are typically important for three-dimensional structural systems, which often have closely spaced frequencies.

As per Uniform Building Code (1994), the largest damping ratio that can be considered when developing site-specific spectra is specified to be 0.05. For this damping value, the cross modal coefficient ρ_{ij} can be estimated directly from the curve in Figure 2 for $\xi = 0.05$. Furthermore, ρ_{ij} can be assumed to be negligible when $r \le 0.67$.

Analysis scheme

For spectral analysis purposes, using different acceleration spectra, only one direction, that is, short direction of the building have been selected. This directional analysis have been performed by locking the global degrees of freedom of other direction. Figure 3 shows the analysis scheme clearly. As spectral acceleration was applied towards the short dimension of the building, the mode shapes transverse to the direction and torsional mode shapes, should have no mass participation factor to the final analysis. To validate this, two analyses have been performed taking 10 and 16 storey special moment resisting concrete frames keeping all the degrees of freedoms unlocked. The mass participation factors for both the analyses considering the whole building, have been listed in Table 2. It is clearly observed from Table 2 that modes perpendicular to the direction of analysis and torsional modes have no participation to final result. Taking this fact into account and to minimise the time of computer run, short direction analysis scheme have been adopted for further analyses, which is expected to produce equally good accuracy of the analytical analysis. Comparing the total mass participation factors as reported, it can be said that total mass participation factor actually increases when the unnecessary modes which have no contribution to the final result, are excluded.

COMPARISON OF TIME PERIOD

Different seismic Codes of practice have been compared with more refined methods of calculating time period using STRAND6 (1996). STRAND6 (1996) uses the following Eqn. to get eigenvalues, which eventually gives the natural frequency of the building.

$$[K]\{x\} = \chi[M\{x\}] \qquad (12)$$

Here, $[K]$ is the banded stiffness matrix, $\{x\}$ is the eigenvector, χ is the eigenvalues and $[M]$ is mass matrix (Consistent and lumped). The building frame shown in Figure 1 has two predominant directions - short and long. Natural time period of the whole building for both the directions has been calculated and plotted in Figure 4. It is observed from Figure 4 that for moment resisting concrete frame these time periods were independent of building direction. For comparison purpose these time periods have been plotted again in Figure 5 with the approximated formulas proposed by various seismic Code provisions. It is clear from Figure 5 that time period calculated using relatively refined method is larger than the Code specified approximate formulas which eventually produce less spectral ordinate in response spectrum analysis. It emphasises the fact that further refinement in the equation pertaining to the calculation of time period may not be rewarding.

COMPARISON OF BASE SHEAR

Efforts have been made to compare the base shear calculated using the Code specified response spectra and response spectra developed by Ansary et al. (1998) using a number of synthetic earthquakes. Response spectrum analysis which has been discussed in a companion paper (Ansary et al., 1998) has been used to calculate the base shear of the moment resisting concrete frame. Once again, STRAND6 (1996) has been used to perform these computations.

To calculate the base shear for different time periods, height of the moment resisting concrete frames have been varied from storey one to storey sixteen. Spectral analysis has been performed for soil type II. Several computer runs were required to get the spectral shapes as plotted in Figure 6. Base shear for a particular building has been found by summing all the horizontal forces of each of the column base for a particular direction. It is observed from Figure 6 that base shear coefficient for the spectra developed by Ansary et al. (1998) is greater than the base shear coefficient produced by Uniform Building Code (1994) at lower time period and decreased faster in higher time periods. It can be concluded by observing Figure 6 that serious thought should be given during future updating of the spectral shapes. Whereas the higher values of spectral ordinates at lower time periods might be left unchanged (as they would lead to more conservative design), faster attenuation at higher time periods, as observed here, may be modified to keep it in-tune with the conservative nature of existing spectra.

COMPARISON OF BASE SHEAR DISTRIBUTION

Base shear distribution has been compared with the corresponding shear distribution proposed by Uniform Building Code (1994). For the purpose of comparison, 16 and 10 storey special moment resisting concrete frames have been selected. In Figure 7 normalised storey shear has been plotted against percent height using UBC (1994) acceleration spectra, proposed simulated acceleration spectra for soil type II. Normalised storey shear distribution proposed by UBC (1994) has also been included in this figure for comparison purpose. It is observed from this figure that storey shear distribution for different spectral analysis have very little difference. The

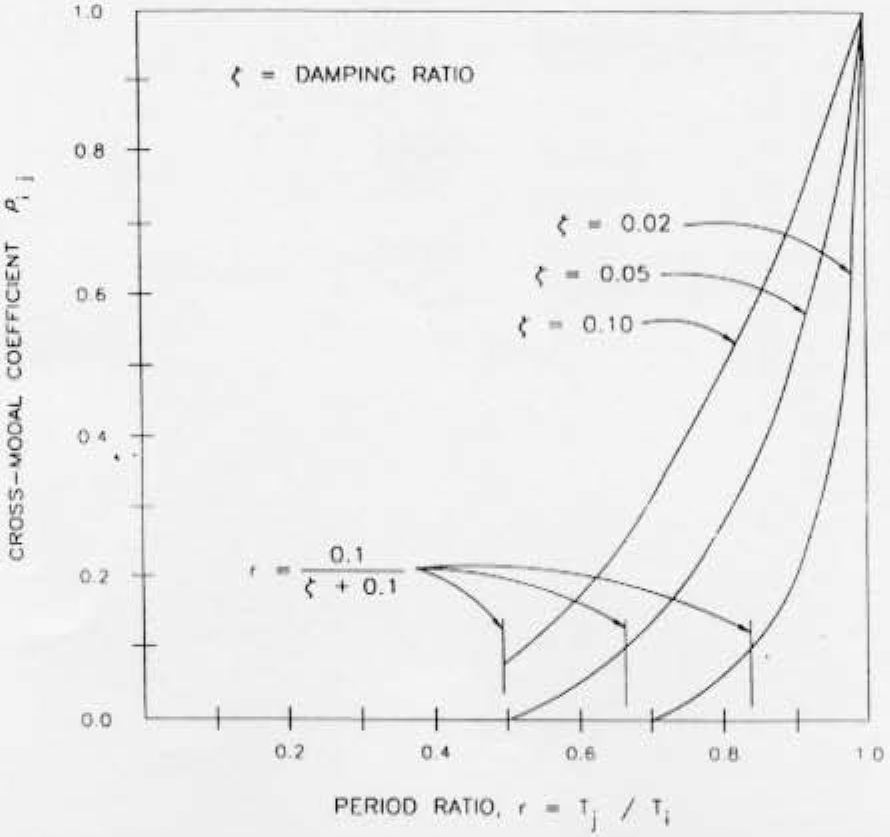


Figure 2. Effect of damping and period ratio on cross-modal coefficient ρ_{ij}

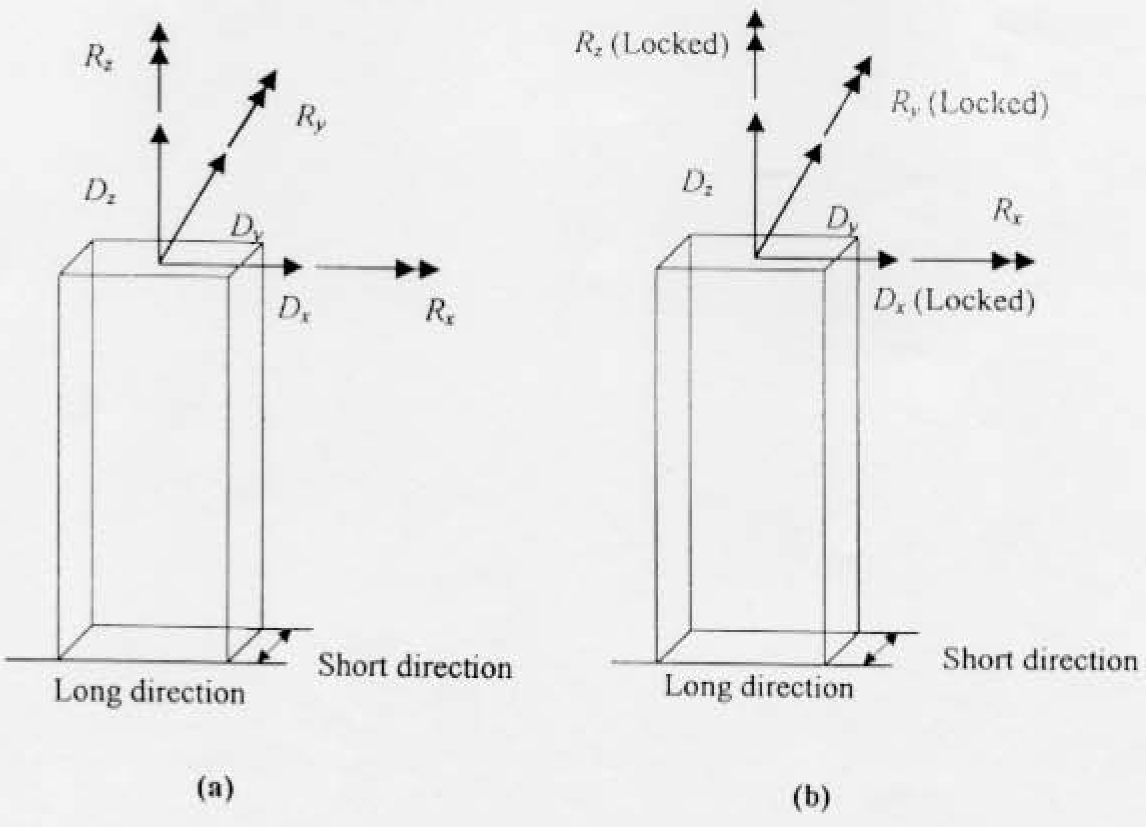


Figure 3. Directional analysis concept (a) whole building analysis
(b) one direction building analysis

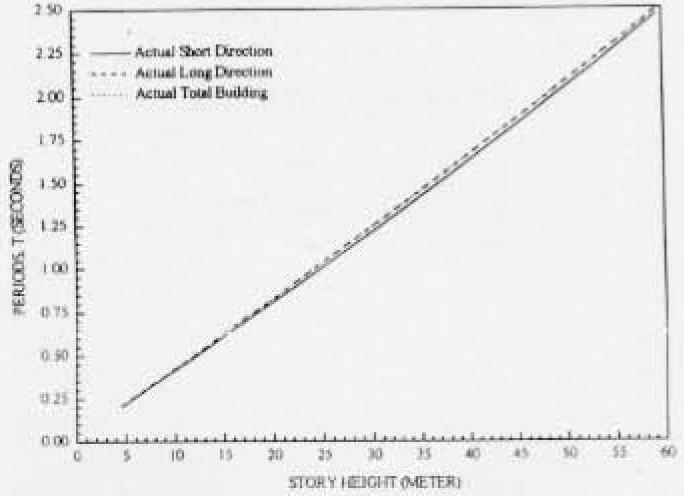


Figure 4. Time period for moment resisting concrete frame considering short, long and total building

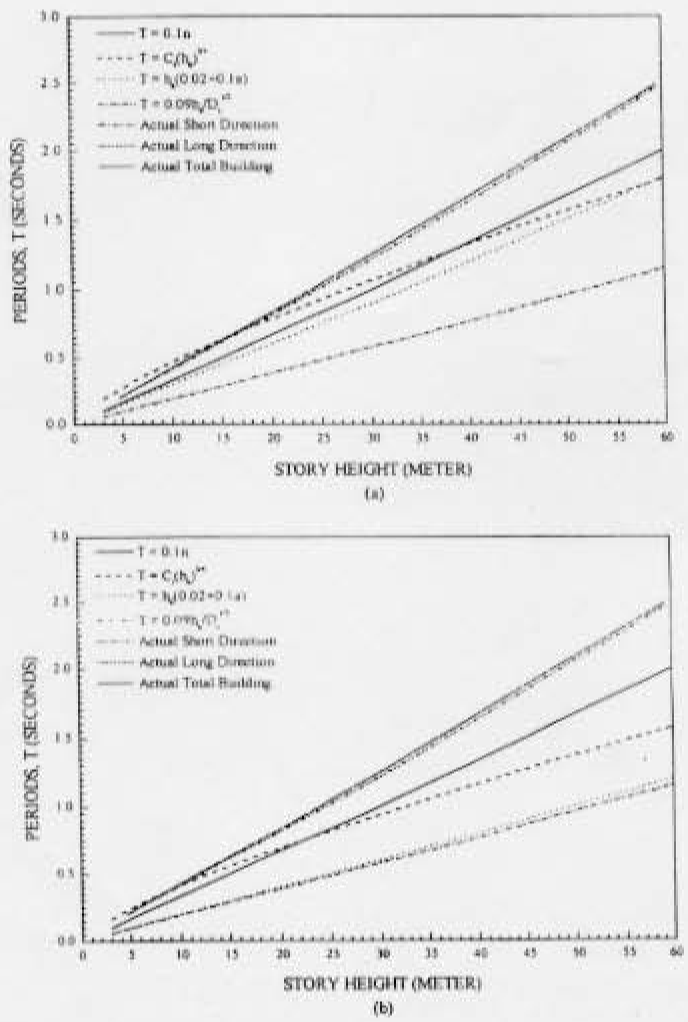


Figure 5. Comparison of more refined method of time period with that of equations proposed by seismic code

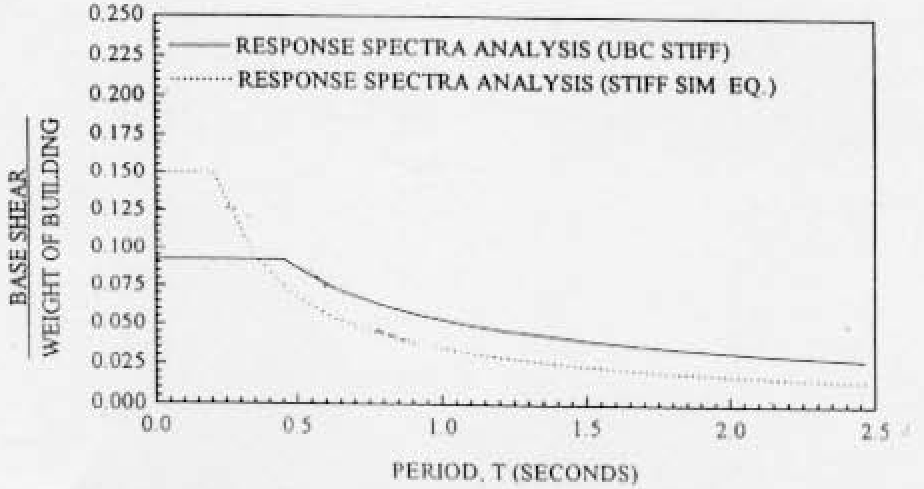
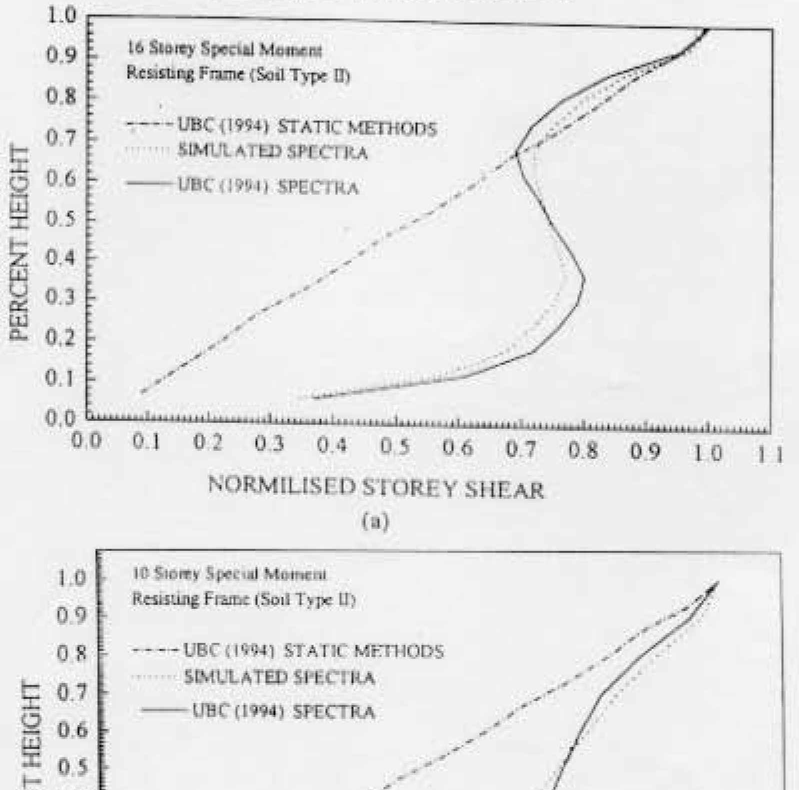
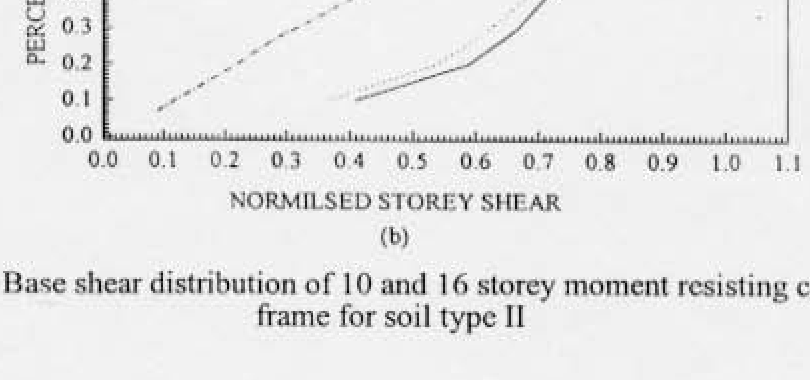


Figure 6. Comparison of base shear coefficient of developed spectra with UBC (1994) for soil type II



(a)



(b)

Figure 7. Base shear distribution of 10 and 16 storey moment resisting concrete frame for soil type II

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static storey shear distribution of UBC (1994) adopt linear formula which is absent in spectral solutions. It can be concluded that, further improvement of storey shear distribution can be made during future changes in seismic Codes.

PROPOSED DESIGN SPECTRA

In a companion paper (Ansary et al., 1998) response spectra for simulated earthquakes have been developed. Further these spectra have been modified and compared with UBC (1994) and have been found to be quite consistent. Efforts have been made here to propose a design response spectra, for 5 percent damping.

Table 3 lists the time period for maximum spectra, maximum spectral ordinate and coefficients a, b of the equation $S = aT^b$ which has been used to modify response spectra for simulated earthquakes. It is observed from the Table 3 that the coefficient b which represents the rate of attenuation is always near or greater than 1, as such the value of b has been fixed to 1. This has been done to remain conservative in higher time period. Higher values at lower time period has been retained as it has found. The final design spectra after this modification for simulated earthquakes is shown in Figure 8. The static equivalent equations have been listed in Table 4 for different soil types derived using simulated earthquakes.

Table 3. Value of Maximum Spectral Ordinate and Coefficient a, b for Modified Simulated Response Spectra for Different Site Category

Site Category	Time Period	Maximum Value of Spectral Ordinate	Coefficient a	Coefficient b
I	0.16 sec.	3.18	0.74	-0.90
II	0.21 sec.	3.30	0.95	-0.93
III	0.35 sec.	3.37	1.48	-0.98
IV	1.10 sec.	2.83	3.74	-1.07

Table 4. Site coefficient derived from simulated earthquake for different site category

Site Category	Site Coefficient
I	$S = 0.51/T$
II	$S = 0.69/T$
III	$S = 1.18/T$
IV	$S = 3.11/T$

CONCLUDING REMARKS

From this study, it is observed that ordinates of response spectra, based on simulated earthquakes, are greater than UBC (1994) spectra at lower time periods and decrease faster in higher time periods. It has been proposed in this study to keep the higher spectral value at lower time periods unchanged. From normalised storey shear and percent height relation, it has been observed that storey shear distribution for different spectral analysis have very little difference. The static storey shear distribution of UBC (1994) adopts linear formula which is absent in spectral solutions. Here, design response spectra for 5 percent damping based on simulated earthquakes for different soil types have been proposed. Faster attenuation has been modified and attenuation rate has been suggested which is inversely proportional to time period.

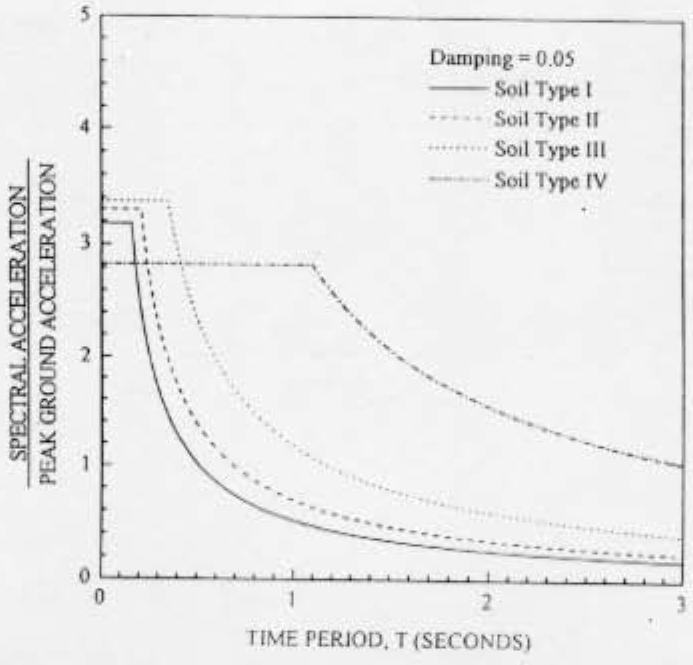

Figure 8. Design spectra based on simulated earthquake for different soil types

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