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Performance of ACI approximate flexible method in the design of mat foundation

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ABSTRACT: While in the past the conventional method of analysis has been found to be inadequate to give realistic results in the analysis of mat foundations, the performance of ACI approximate flexible method could not be checked thoroughly due to the complex nature of the method as well as unavailability of suitable computer programs catering for the method. In the present study an easy to use computer program, capable of analysing mat foundations using ACI Approximate flexible method, has been employed. The program gives thickness of mat calculated on the basis of safety against punching. The output gives moments, shears and deflections at points whose co-ordinates are specified in the input. The computer program for ACI approximate flexible method, along with available standard computer programs for discrete element methods like finite grid method and finite difference method, has been used in this study in order to ascertain the performance of ACI approximate flexible method with respect to other popular methods.

1 INTRODUCTION

Mats can be analysed using Conventional method, ACI approximate flexible (ACIA) method, finite difference (FD) method, and finite grid method with and without soil spring zoning (FGZ and FG). All but the conventional method utilise, in the analysis procedure, modulus of subgrade reaction, which is in turn related to the soil pressure and its deformational characteristics. Whereas, in the FG method, the soil springs remain uncoupled i.e. deformation of one spring is not affected by others, the soil spring zoning technique allows FGZ method to cater for the coupling action of springs by employing softer springs towards the centre. A detailed description of mat analysis techniques is available elsewhere in Bowles (1974, 1986, 1988), ACI 436 (1966) and Shukla (1984). However, a brief account of the ACI approximate flexible method will be presented here since the performance of this method will be tested against other more refined methods as well as the conventional method in this study.

2 ACI APPROXIMATE FLEXIBLE METHOD

ACI Committee 436 (1966) recommended this method for the general case of a flexible mat supporting columns at random locations with varying intensities of load. This procedure is based on theories of circular plate on Winkler medium.

The effect of a concentrated load on a typical mat has been found to be damped out quite rapidly. It is, therefore, possible to consider the mat as a plate and determine the effect of a column load in the area surrounding the load. By superimposing the effect of all the column loads within the zone of influence, the total effect of all the column loads at any point is determined. This zone of influence is generally not large and it is suggested that it is not necessary to consider columns further than two bays in all directions to determine stresses at a particular point in most cases. Since the effect of each load is transmitted through the mat in a radial direction, the use of polar co-ordinates is necessary.

Initially, the overall thickness of mat is calculated from shear requirements at critical section, (ACI 318-89). Then the modulus of subgrade reaction (k_s) is established. The flexural rigidity D of the mat in lb-inch is calculated using Equation 1.

$$D = \frac{E_c t^3}{12(1-\mu^2)} \quad (1)$$

where, E_c, the modulus of elasticity of concrete, is given by $57000\sqrt{f'_c}$, (in psi), t is thickness of mat (in inch) and μ is Poisson's ratio of concrete (0.15 to 0.25).

The radius of effective stiffness L is then calculated as follows:

$$L = \sqrt[4]{\frac{D}{k_s}} \tag{2}$$

The radius of influence is taken as $4L$.

The radial and tangential moments (M_r, M_t) and the shear (Q) and deflection (w) at a point are calculated using the following equations:

$$M_r = -\frac{p}{4}\left[Z_4(\tfrac{r}{L}) - (1-\mu)\frac{Z_3'(\tfrac{r}{L})}{\tfrac{r}{L}}\right] \tag{3a}$$

$$M_t = -\frac{p}{4}\left[\mu Z_4(\tfrac{r}{L}) + (1-\mu)\frac{Z_3'(\tfrac{r}{L})}{\tfrac{r}{L}}\right] \tag{3b}$$

$$Q = -\frac{p}{4L} Z_4'(\frac{r}{L}) \tag{3c}$$

$$w = \frac{pL^2}{4D} Z_3(\frac{r}{L}) \tag{3d}$$

where, r is distance of point under investigation from point load along radius, p is concentrated load, Z_3, Z'_4, etc. are functions first introduced by Schleicher(1926) and can be found in Table III of Hetenyi (1946).

In order to convert radial and tangential moments to rectangular co-ordinates (M_x, M_y), the following equations are used:

$$M_x = M_r \cos^2\phi + M_t \sin^2\phi \tag{4a}$$

$$M_y = M_r \sin^2\phi + M_t \cos^2\phi \tag{4b}$$

Here, ϕ is the angle of the inclination of the point with x-axis. For the combination of responses due to all the column loads, the effects of individual columns should be superimposed at points of interest. Again, if the edge of the mat is at a distance less than $4L$ from an individual column load, a correction should be applied as follows:

a) Moment, shear, and deflection at the edge of the mat due to column loads within the radius of influence should be calculated as stated above.

b) The mat may be divided into strips of unit width in both directions.

c) Assuming the strips as semi-infinite beams, shear and moment equal and opposite to those obtained in step (a) should be applied and their effects at various points superimposed on the respective values obtained earlier.

d) For moment, shear and deflection in a semi infinite beam, the following relationships are used:

$$M_x = M_1 A_{\lambda x} - \frac{P_1}{\lambda} B_{\lambda x} \tag{5a}$$

$$Q_x = -2M_1 \lambda B_{\lambda x} - P_1 C_{\lambda x} \tag{5b}$$

$$w_x = -\frac{2M_1\lambda^2}{k_s} C_{\lambda x} + \frac{2P_1\lambda}{k_s} D_{\lambda x} \tag{5c}$$

where, M_1, P_1 are moment and shear, respectively from step (a).

$$\lambda = \sqrt[4]{\frac{k_s b}{4E_c I_b}} \tag{6}$$

b = width of mat strip, 1 ft.
I_b = moment of inertia of mat strip.

$$A_{\lambda x} = e^{-\lambda x}(\cos\lambda x + \sin\lambda x) \tag{7a}$$

$$B_{\lambda x} = e^{-\lambda x}\sin\lambda x \tag{7b}$$

$$C_{\lambda x} = e^{-\lambda x}(\cos\lambda x - \sin\lambda x) \tag{7c}$$

$$D_{\lambda x} = e^{-\lambda x}\cos\lambda x \tag{7d}$$

Again, the Z functions have the characteristic features of exponential waves; Z_1 and Z_2 increase rapidly with the increase of their argument while Z_3 and Z_4 decrease as the argument ($x = r/L$) increases as shown in Figure 1. They can be written in the form of power series as follows:

$$Z_1(x) = 1 - \frac{(x/2)^4}{(2!)^2} + \frac{(x/2)^8}{(4!)^2} - \frac{(x/2)^{12}}{(6!)^2} + .. \tag{8a}$$

$$Z_2(x) = -\frac{(x/2)^2}{(1!)^2} + \frac{(x/2)^6}{(3!)^2} - \frac{(x/2)^{10}}{(5!)^2} + ... \tag{8b}$$

$$Z_3(x) = \frac{1}{2}Z_1(x) - \frac{2}{\pi}[R_1(x) + Z_2(x)\log\{\gamma^* \frac{x}{2}\}] \tag{8c}$$

$$Z_4(x) = \frac{1}{2}Z_2(x) + \frac{2}{\pi}[R_2(x) + Z_1(x)\log\{\gamma^* \frac{x}{2}\}] \tag{8d}$$

where,

$\log\gamma^* = C_E$ = Euler's constant = 0.577216

$$R_1(x) = (x/2)^2 - \frac{\phi(3)}{(3!)^2}(x/2)^6 + \frac{\phi(5)}{(5!)^2}(x/2)^{10} - .. \tag{9a}$$

$$R_2(x) = \frac{\phi(2)}{(2!)^2}(x/2)^4 - \frac{\phi(4)}{(4!)^2}(x/2)^8 + \frac{\phi(6)}{(6!)^2}(x/2)^{12} - .. \tag{9b}$$

$$\phi(n) = \sum_{i=1}^{n}\frac{1}{i} \tag{9c}$$

3 THE COMPUTER PROGRAMS

The program for FD method has been suitably adopted for microcomputers from Bowles (1974). The program for FG and FGZ methods is taken from Bowles (1988) after modification. Data generation programs have also been developed for automation of these programs (Hossain, 1993). A computer program is developed and tested for the ACIA method. Details of the program along with tests for validity is available elsewhere in Hossain (1993).

4 THE TRIAL MAT

A trial mat is selected for using it in all the analyses presented in this study. A building supported on mat is selected to carry out the comparative study. Figure 2 shows the plan of the mat with design load on each column. The mat is 72 ft. (21942 mm) by 60 ft. (18285 mm) in plan dimensions. The building is 7 storied having each storey 12 ft. (3658 mm) high. The mat is 30 inch (762 mm) thick which is safe against punching due to column loads used in this study.

Dead and live loads are determined on the basis of a flat plate having 9 inch (229 mm) thickness and a live load of 80 psf (3.83 kN/m^2). A reduction in factored live load as per Winter and Nilson (1981) is used. It has been found that for the 7 storied building, the loads due to wind is very small compared to dead and live loads and combination of dead and live load governs the mat design.

The most important soil property required in the analysis is the modulus of subgrade reaction (k_s). A value of 100 kcf (15709 kN/m^3) is taken for soil having allowable bearing capacity q_a = 3 ksf (144 kN/m^2) which is reasonable for Dhaka city, considering a factor of safety of 3. The other soil parameter required is the maximum linear soil deflection which is assumed to be 19 mm according to Bowles (1988). The cylinder strength, unit weight and Poisson's ratio of concrete are taken as 3000 psi (20.7 MPa), 0.15 kcf (23.6 kN/m^3) and 0.15, respectively.

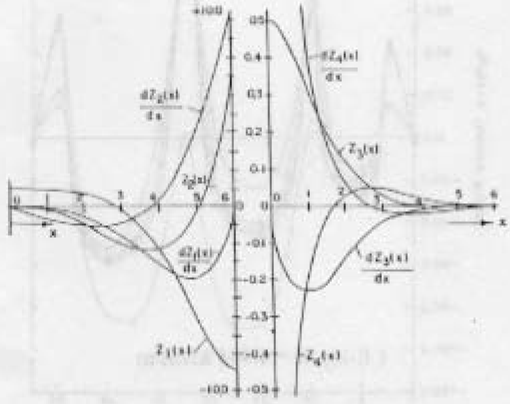

Fig. 1 Variation of Z functions with x (After Hetenyi, 1946)

5 THE COMPARATIVE STUDY

5.1 *Moments along column lines*

The variation of moment along column lines A, C, 1 and 3 are presented in Figures 3 to 6, respectively. Here, results obtained from all the five different approaches FGZ, FG, FD, Conventional and ACIA method are plotted. A summary of moment values at various points along column lines A, C, 1 and 3 are also given in Table 1.

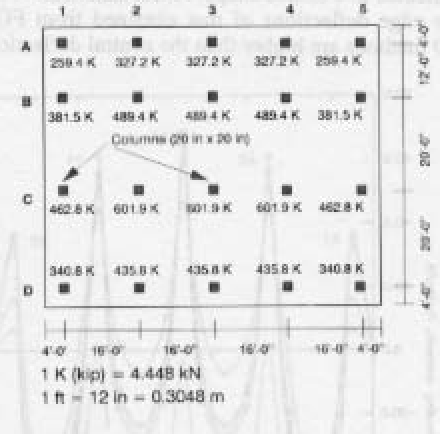

Fig. 2 Mat dimensions with column loads

It is clear that the ACIA method yields results more or less comparable to discrete element methods at points far from the edges and compares less favourably near the edges. This method is based on functions derived for infinite plates. Although end conditioning forces are applied to simulate a finite mat, they are applied in strips and cause concentrated effects at the edges.

It is notable that the FD method, which is based on the basic differential equation of plates resting on Winkler's medium, has been found to yield results comparable to the FG and FGZ methods, which utilise the techniques of finite element.

By comparing the (positive) moments at column points obtained by different methods, it has been found that the maximum moment is due to FGZ method. Next higher values are yielded by FG and FD methods. The ACIA method gives least value among comparable methods. As for (negative) moments between column points, the maximum moment among comparable methods is due to FG method. The FGZ, FD and ACIA methods yield next higher moments, respectively.

5.2 *Deflections along column lines*

Deflections obtained by different methods along column lines A and C, and 1 and 3 are presented in Figures 7 and 8, respectively. Deflection curves show that FG and FD methods give almost equal deflections at all the points; both these methods consider uniform soil springs under the mat. The results of FGZ clearly indicate the effect of zoning.

In this method, the deflections of interior points are much higher than those of FG and FD methods. At column lines A and 1, the deflections at the edges are lower for FGZ in comparison FG. This may be visualised as the uplift of a corner of plate which is deflected in a dished shape. At column lines A and C the edge deflections of mat obtained from FG and FD methods are higher than the central deflections.

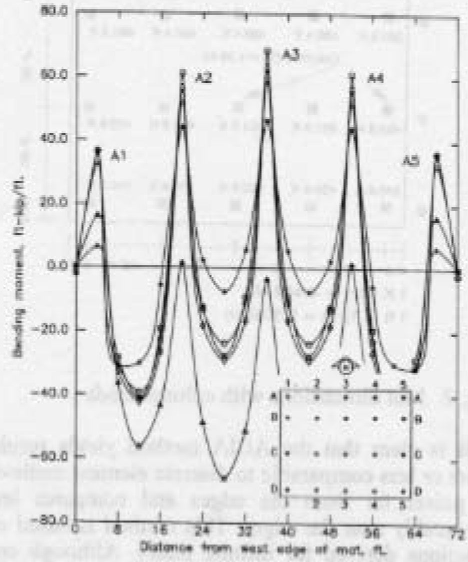

Fig. 3 Comparison of moments along column line A

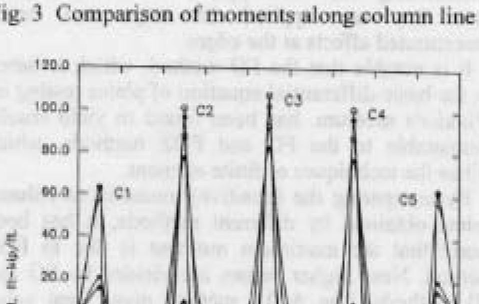

Fig. 4 Comparison of moments along column line C

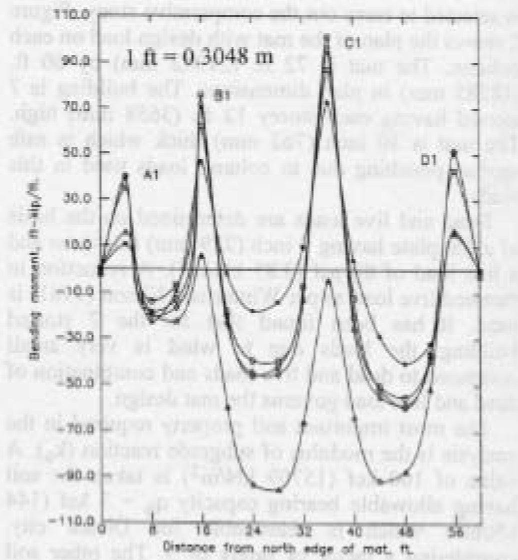

Fig. 5 Comparison of moments along column line 1

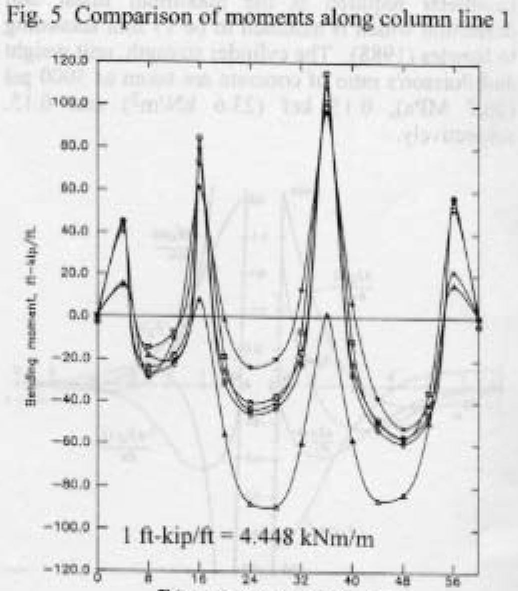

Fig. 6 Comparison of moments along column line 3

The ACIA method yields very high deflections at all the edges due to the concentrated effect of end conditioning forces. At exterior column lines, the deflection of interior columns are quite small compared to other methods. When deflections at line A is calculated, the mat is first assumed to be infinite in all directions and to correct this assumption, end conditioning forces are applied at east and west edges considering a beam of unit width. But the mat still remains infinite on north edge; as a result the deflections of inner columns along column line A and 1 are small. For interior column lines, the ACIA deflection of points away from edges are comparable to FG and FD methods.

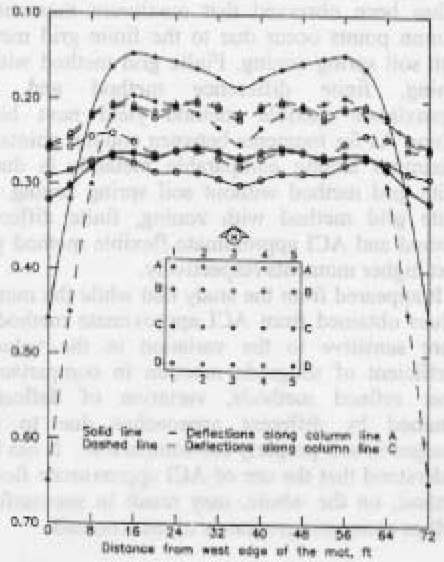

Fig. 7 Comparison of deflections along column lines A and C

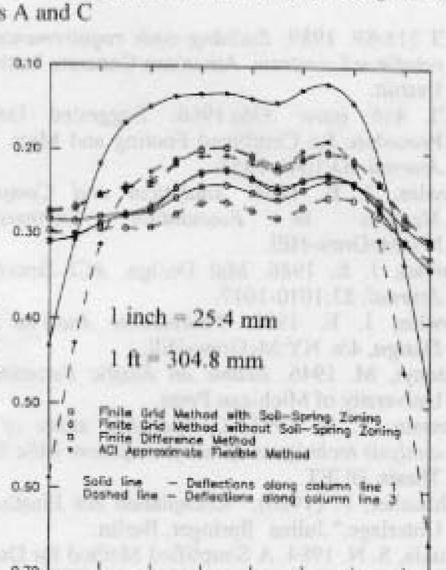

Fig. 8 Comparison of deflections along column lines 1 and 3

Table 1. Comparison of moments (ft-kip/ft), obtained by different methods along lines A, C, 1 and 3. (1 ft-kip/ft = 4.448 kNm/m)

Point	FGZ	FD	ACIA
A1 (Line A)	36.47 (100.0 %)	32.65 (89.5 %)	6.44 (17.6 %)
A2 (Line A)	61.13 (100.0 %)	52.27 (85.5 %)	44.17 (72.3 %)
A3 (Line A)	68.60 (100.0 %)	59.06 (86.1 %)	46.44 (67.7 %)
A1-A2 (Line A)	-39.44 (100.0 %)	-41.44 (105.1 %)	-30.21 (76.5 %)
A2-A3 (Line A)	-23.85 (100.0 %)	-27.80 (116.6 %)	-7.53 (31.6 %)
C1 (Line C)	62.55 (100.0 %)	57.14 (91.4 %)	20.05 (32.1 %)
C2 (Line C)	100.88 (100.0 %)	87.39 (86.6 %)	77.26 (76.6 %)
C3 (Line C)	107.51 (100.0 %)	92.78 (86.3 %)	82.67 (76.9 %)
C1-C2 (Line C)	-50.32 (100.0 %)	-44.47 (88.4 %)	-48.99 (97.4 %)
C2-C3 (Line C)	-37.04 (100.0 %)	-33.45 (90.3 %)	-16.82 (45.4 %)
A1 (Line 1)	39.94 (100.0 %)	34.94 (87.5 %)	10.46 (26.5 %)
B1 (Line 1)	75.77 (100.0 %)	62.00 (81.8 %)	46.58 (61.5 %)
C1 (Line 1)	101.14 (100.0 %)	86.73 (85.8 %)	72.66 (71.8 %)
D1 (Line 1)	51.42 (100.0 %)	42.99 (83.6 %)	17.17 (33.4 %)
A3 (Line 3)	44.99 (100.0 %)	40.72 (90.5 %)	14.22 (31.6 %)
B3 (Line 3)	84.41 (100.0 %)	73.04 (86.5 %)	61.37 (72.7 %)
C3 (Line 3)	115.02 (100.0 %)	101.46 (88.2 %)	97.10 (84.4 %)
D3 (Line 3)	57.08 (100.0 %)	51.89 (90.9 %)	56.30 (98.6 %)

5.3 *Sensitivity to variation in k_s*

To find the sensitivity of the analysis methods due to a change in k_s, the modulus is varied from 50 to 300 kcf (7855 to 47127 kN/m^3) keeping all other geometric and material properties unchanged.

The moments M_X (moment about north-south line) at column point A1 obtained by FG, FGZ, FD and ACIA methods for different modulus of subgrade reaction of soil are presented in Figure 9. These methods give 2.5%, 3.5%, 3.9% and 92.9% higher values for an increase in the magnitude of soil modulus from 50 kcf to 300 kcf. The results of ACIA method is not at all comparable due to the concentrated effect of end conditioning forces at this corner column.

For column point C3, moments M_x are presented in Figure 10. Here, FG and FD methods yield 1.1% and 3.3% higher moments for 300 kcf than for 50 kcf. But FGZ and ACIA methods yield 6.1% and 20.3% less moment for the same change in k_s.

For column point A3, deflections are shown in Figure 11. For a modulus of 300 kcf, FGZ, FG, FD and ACIA methods yield deflections 82.0 %, 81.7 %, 81.3 % and 76.7 % less than for a modulus of 50 kcf.

In general, the effect of modulus of subgrade reaction on deflection of mat is almost similarly portrayed by all the methods.

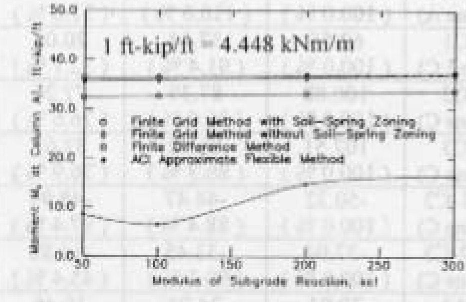

Fig. 9 Variation of M_x at A1 with k_s

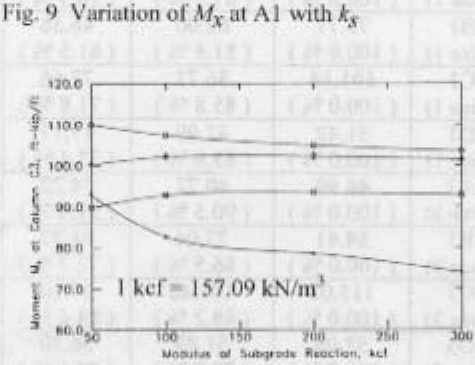

Fig. 10 Variation of M_x at C3 with k_s

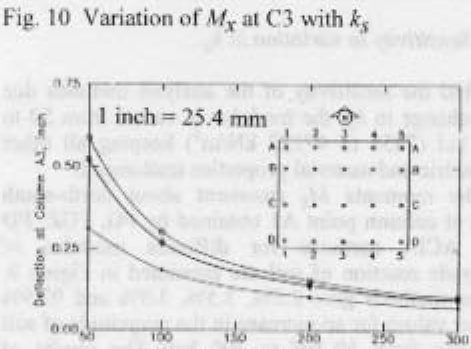

Fig. 11 Variation of deflection at A3 with k_s

6 CONCLUSIONS

It has been revealed that while ACI approximate flexible method yields moments and deflections comparable to discrete element methods at points away from the edges, in the vicinity of edges, the method has been found to be somewhat inadequate. It has been observed that maximum moments at column points occur due to the finite grid method with soil spring zoning. Finite grid method without zoning, finite difference method and ACI approximate flexible method yield next higher values. As for moments between column points, the maximum among comparable methods is due to finite grid method without soil spring zoning. The finite grid method with zoning, finite difference method and ACI approximate flexible method yield next higher moments respectively.

It appeared from the study that while the moment values obtained from ACI approximate method are more sensitive to the variation in the value of coefficient of subgrade reaction in comparison to other refined methods, variation of deflections obtained by different approaches due to such changes in soil property are comparable. It has been understood that the use of ACI approximate flexible method, on the whole, may result in successful as well as economic prognosis of mats on soil.

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