LOAD-DISPLACEMENT RELATIONSHIP OF SQUARE FOOTINGS ON DHAKA SOIL

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ABSTRACT

In this study, the critical state finite element program CRISP has been employed in predicting the load-displacement behaviour of square footings resting on Dhaka soil at various depths. Square footings have been analyzed as an axisymmetric case by idealizing them as circular footings having the same equivalent area as their square footing counterpart. The interface of footing and soil has also been modeled. The clay has been considered to follow Modified Cam-clay constitutive law, while sand layer follows elastic-perfectly-plastic material properties. After conducting an extensive parametric study, the load-displacement (P-δ) relationship of square footings has been found to be related by a hyperbolic function, $P = (A\delta/(B+\delta))$; A and B being constants. The ensuing load-displacement equation traced the finite element predictions well. Apart from assessing differential settlements, which may be conveniently used as inputs in the analysis of framed structures, footing sizes and depths may be chosen, albeit approximately, using the equation developed, *via* settlement equalization of footings.

KEYWORDS

Dhaka, soil, Footing, Load-displacement, CRISP, Cam-clay.

INTRODUCTION

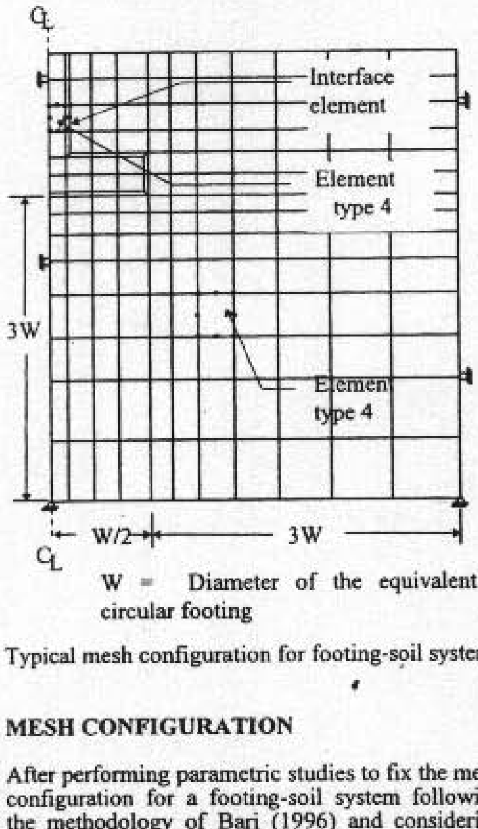
After making an extensive study on pile-soil system (Bari, 1996), attempts have been made in this study to use the FE model to predict the load-displacement behaviour of square footings on Dhaka soil. Square footings have been analyzed in this study as an axisymmetric case. In doing so square footings have been idealized as circular footings having the same equivalent area as their square footing counterpart. Although this idealization is expected not to change the responses of square footing significantly, it simplifies the analysis to a great extent. In contrast to axisymmetric idealization, if a plain-strain idealization would have been adopted, the dimension of the model footing in the transverse plane would have assumed an infinite length. This would have invariably turned a square footing into a strip footing. Thus, use of axisymmetric modelling has been considered to be adequate, although not as an alternative to a fully three-dimensional study, which is beyond the scope of the present work. Again, the axisymmetric idealization of square footing simulates the mode of load transfer from column to footing and from footing to soil in a way comparable to the mode experienced in case of square footings.

As in the case of piles, the soil and footing elements have been analyzed in this study as linear strain quadrilateral with displacement unknown (type 4) and interface of footing and soil as the 6 noded interface elements with displacement unknown (see Figure 1). The clay has been considered to follow Modified Cam-clay (MCC) constitutive law, while sand layer follows elastic-perfectly-plastic material properties. Drained analysis, instead of undrained or consolidation analysis, has been performed during the study of footing-soil interaction. Actually, a consolidation analysis with larger time span might have predicted the responses more realistically. On the other hand, if consolidation analysis is allowed to undergo for longer time period, then the displacement prediction from consolidation analysis converge to the displacement prediction from drained analysis.

Thus, the drained analysis usually predicts higher displacements, which are not far different from the displacement predicted by consolidation analysis with sufficiently long time period. As a result, the drained analysis has been performed in this study and thus, the escalating running time cost that would have been incurred in case of consolidation analysis has been averted.

It is expected that the studies to be described here will lead to displacement predictions for footings on Dhaka soil under various conditions. Once displacements of all the footings are known, relative displacements among various footings can be determined. These relative displacements may be given as inputs to frame analysis in order to arrive at a more realistic prognosis.

Figure 1



W = Diameter of the equivalent circular footing

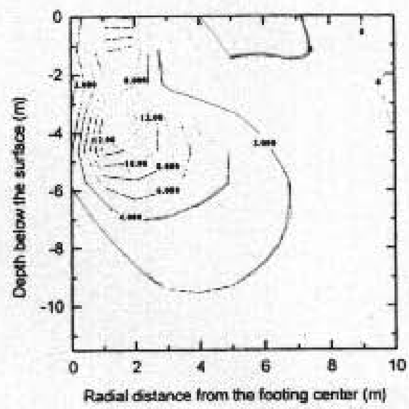
Typical mesh configuration for footing-soil system

MESH CONFIGURATION

After performing parametric studies to fix the mesh configuration for a footing-soil system following the methodology of Bari (1996) and considering the mesh configurations for footing presented by Dewaikar and Prajapati (1992) and Kaliakin and Li (1995), a representative mesh configuration, as shown in Figure 1, has been chosen for footing-soil system in this study.

Besides, the shear stress contour for the above mentioned footing has been plotted and shown in Figure 2. It shows that the stress zone is well within the mesh boundaries and high stress zone occurs near the bottom edge of the footing. Thus, the mesh configuration adopted which has finer mesh near the footing corner and reasonable extent of horizontal and vertical boundaries, appears to be an acceptable selection.

Figure 2



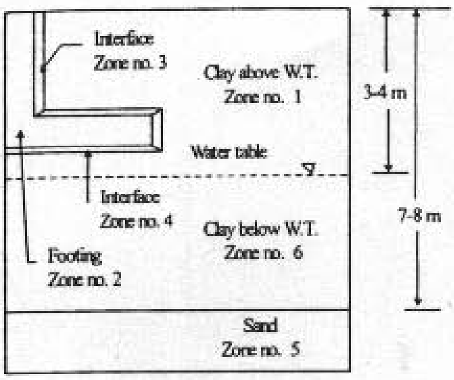
Shear stress contour for footing

MATERIAL CHARACTERISTICS

As this study mainly aims at determining a design rationale to predict load-displacement behaviours of square footings buried in Dhaka soil, a representative soil profile of Dhaka soil has been considered. In doing so, the average values of different soil parameters have been selected by considering a number of soil investigation reports available for Dhaka soil. Although the soil conditions in some parts of Dhaka may differ slightly from the parameters considered in this study, it should be kept in mind that the soil parameters selected in this study represent average Dhaka soil conditions. Moreover, this study is mainly concerned with proposing a methodology by which the load-displacement responses for any soil type can be formulated. The deviation of actual footing displacement from model footing displacement due to the use of average soil properties is expected to play a not so important role in the input to the design of a superstructure, where, in fact, relative displacement is expected to play a more important role.

The representative soil profile considered in this study has been presented in Figure 3. The figure shows that the water table has been assumed to be at a depth of 3 to 4 m from the soil surface. In the event the water table rises above this level, it is expected to reduce effective *in-situ* pressure slightly resulting in a slight increase in the footing displacement. Besides, the material properties of different material zones (see Figure 3) have also been presented in Tables 1, 2, 3 and 4.

Figure 3



Representative soil profile for typical Dhaka clay

TABLE 1
PARAMETERS FOR REPRESENTATIVE CLAY LAYER OF DHAKA

Soil Type	Zone no	κ	λ	e_{cs}	M	ν	γ_{bulk} (kN/m^3)
Clay > W.T	1	0.02125	0.085	1.08	0.90	0.25	14.5
Clay < W.T.	6	0.02125	0.085	1.08	0.90	0.25	19.5

TABLE 2
PARAMETERS FOR REPRESENTATIVE SAND LAYER OF DHAKA

Zone No	E_o (kN/m^2)	ν	C (kN/m^2)	ϕ (degree)	γ_{bulk} (kN/m^3)	Rate m_1 (kN/m^2)/m
5	50E3	0.25	0	31	20.0	2.0E3

It should be observed that the interface elements in the back-filled clay layer have been given no shear resistance as has been suggested by Terzaghi (1943) in his shallow foundation theory.

The *in-situ* stresses for different layers have to be calculated using Wroth's (1975) method considering overconsolidated clay for Dhaka (Bari, 1996). From a number of consolidation tests i.e., ($\log_{10}\sigma_v$, e) plot for Dhaka, it has been observed that the overburden pressure on the surface of Dhaka clay has an average value of 50 kN/m^2. The *in-situ* stresses and p_c' have been calculated using this overburden pressure in this study.

TABLE 3
INTERFACE ELEMENT PARAMETERS FOR DHAKA CLAY

Zone No.	C (kN/m^2)	ϕ (deg)	K_n (kN/m^2)	G_s (kN/m^2)	G_{res} (kN/m^2)
3	5	23	23.34 E4	1.01 E4	10
4	0	0	23.34 E4	1.01 E4	10

TABLE 4
PARAMETERS FOR FOOTING MATERIAL

E (kN/m^2)	Zone Number	ν	γ_{bulk} (kN/m^3)
30 E6	2	0.20	23.5

LOAD-DISPLACEMENT RESPONSES

To investigate the effect of variation in the size of the footing or the depth of the footing embedment, a scheme has been followed in this study. Firstly, the depth of the footing embedment has been kept constant (D_F = 2.5 m) and the load displacement responses for different sizes of footing have been investigated which are shown in Figure 4. Next, keeping the size of the footing constant (S_F = 2.5), the depths are varied and the load-displacements responses for them are obtained and plotted in Figure 4.

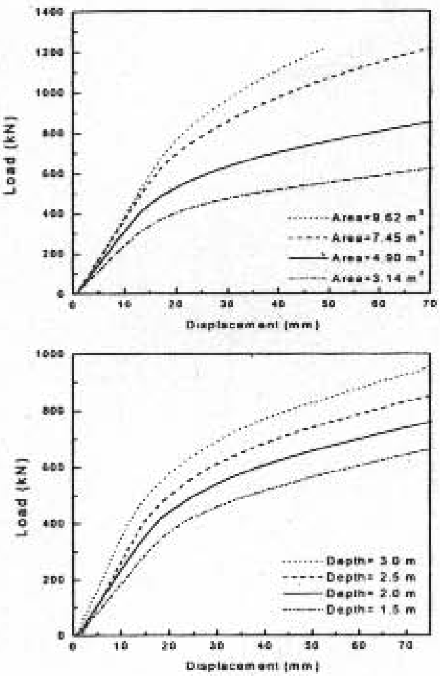
Now, efforts have been made to formulate a general trend of these load-displacement curves in this study. In doing so, a hyperbolic function in the form of Eqn. 1 has been selected after many trials and considerations.

$$P = \frac{A\delta}{B+\delta} \quad (1)$$

where P = load applied on the footing, δ = displacement, A and B are constants.

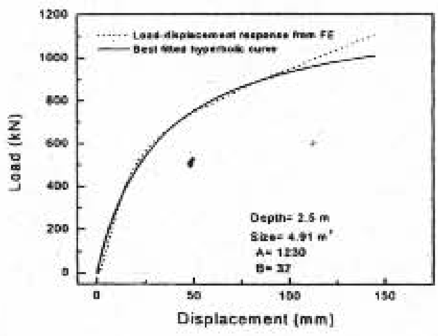
It has been found that Eqn. 1 can trace the actual load-displacement curves reasonably well for significant distance even into the non-linear portion. Only, the portion of curves far away from the point of commencement of non-linearity may deviate considerably from the curves formulated using Eqn. 1. Figure 5 shows a typical load displacement response of a particular footing along with the best fitted curve formulated using Eqn. 1. It is clear from Figure 5 that the best fitted curves using Eqn. 1 simulate the actual tried of load-displacement curves satisfactorily.

Figure 4



Load displacement responses for different (a) areas and (b) depths of footing

Figure 5



Best fitted hyperbolic curve

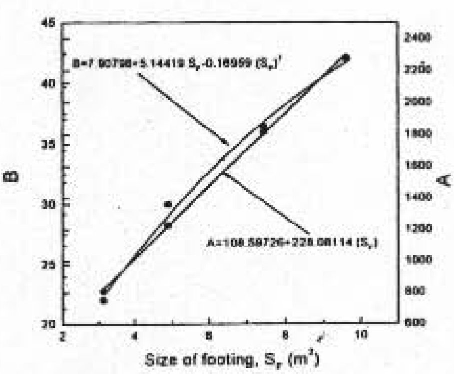
Accordingly, every load-displacement curves shown in Figure 4 has been formulated as a hyperbolic function (Eqn. 1) and different values of constants A and B for various depths and sizes have been found out are presented in Table 5.

Now, the variation of constants A and B with different values of sizes of footing have been shown in Figure 6. Figure 6 shows that the constant A increases linearly with higher sizes. The equation of the best fitted straight lines has also been shown in Figure 6. Similarly, the variation of constant B assumes a parabolic trend for higher sizes as shown in Figure 6 which also includes the equation of the best fitted second degree polynomial.

TABLE 5
VALUES OF CONSTANTS A AND B FOR DIFFERENT SIZES AND DEPTHS OF FOOTING

Size of footing (m^2)	Depth of embedment (m)	A	B
3.14	2.5	810	22
4.91		1230	30
7.45		1850	36
9.62		2275	42
4.91	1.5	1050	42
	2.0	1130	35
	2.5	1230	30
	3.0	1280	26

Figure 6



Variation of constants A and B for different sizes of footing

In the same way, the trend followed by the constants A and B with variation in depths of embedment have been investigated in Figure 7. Figure 7 also include the equations of the best fitted curves.

A PROPOSED LOAD-DISPLACEMENT RATIONALE FOR SQUARE FOOTINGS IN DHAKA

As the constants of Eqn. (1) appear to follow some defined trend with variation of depths and sizes of footings in Dhaka soil, an empirical equation can be introduced following a systematic nonlinear regression analysis. The ensuing empirical equation has been presented below as Eqn. 2.

$$P = \frac{\left(\frac{A_S A_D}{K_A}\right)\delta}{\left(\frac{B_S B_D}{K_B}\delta\right)} \text{ kN} \tag{2}$$

Where

$$K_A = 1230 \text{ and } K_B = 30$$

$$A_S = 108.60 + 228.08(S_F) \tag{3}$$

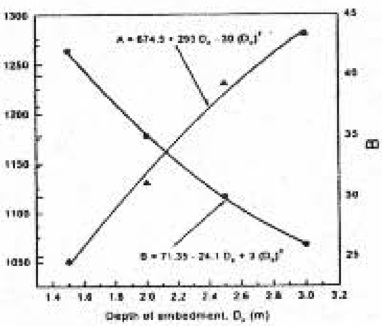
$$B_S = 7.91 + 5.14(S_F) - 0.17(S_F)^2 \tag{4}$$

$$A_D = 674.50 + 293.00(D_F) - 30.00(D_F)^2 \tag{5}$$

$$B_D = 71.35 - 24.10(D_F) + 3.00(D_F)^2 \tag{6}$$

It should be kept in mind that Eqn. (2) is empirical in nature and valid for a certain range of footing dimensions. So proper consideration should be given to the units used and the range for which it is expected to work satisfactorily. Table 6 presents the units and the range of S_F and D_F applicable to Eqn. 2.

Figure 7



Variation of constants A and B for different embedment depths of footing

VALIDATION OF THE PROPOSED METHOD

To show the acceptability of the proposed equation, the load-displacement behaviours predicted by Eqn. 2 has been compared with the corresponding load-displacement behaviours obtained from finite element analysis using the presently used FE model (CRISP). For the purpose of comparison, three examples are used whose parameters have been selected arbitrarily within the range of the equations. The values of necessary parameters for these examples are listed in Table 7.

TABLE 6
UNITS AND RANGE OF SIZES AND DEPTHS OF FOOTINGS FOR EQN. 2

	Unit	Range
Depth	m	15.0-3.00
Size	m^2	3.15-9.62

For these examples, values for constants A and B have been calculated using Eqns. 3, 4, 5 and 6. Now, using these values, a load-displacement equation for each example has been obtained, using Eqn. 2 and shown in Table 7. In addition to that, finite element analysis has been performed separately for each of the example footing cases, and load-displacement responses obtained from these analyses have been compared with the proposed equations.

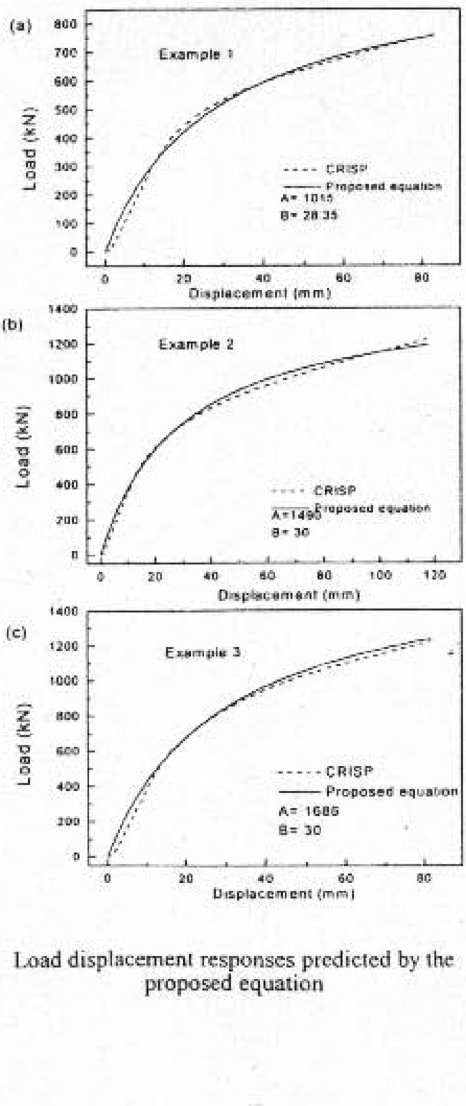
TABLE 7
EXAMPLE FOOTING SIZES AND DEPTHS OF EMBEDMENT

Ex.	Size (S_F) (m^2)	Depth of embedment (D_F) (m)	Equation of load-displacement curve from Eqn. 2
1	4.15	2.25	$P = 1015\delta/(28.35+\delta)$
2	5.94	2.75	$P = 1490\delta/(30.00+\delta)$
3	6.61	3	$P = 1686\delta/(30.00+\delta)$

Figure 8 shows the load-displacement curves obtained from both CRISP and the proposed method in a single plot for all the three examples studied. It is clear from Figure 8 that the load-displacement curves obtained from the proposed method and the load-displacement responses obtained from CRISP are almost the same. Very insignificant deviations which are apparent from Figure 8 can be neglected as far as practicality is concerned. Thus, it can be stated that the proposed empirical equation based on soil-structure interaction study simulates the load-displacement responses satisfactorily for square footings embedded in Dhaka soil.

One thing should be kept in mind that this empirical method can not be applied readily to any site in Dhaka as the site concerned may have different local soil characteristics, which may be widely different from the representative soil characteristics considered in this study for Dhaka soil. But, if the soil conditions of the site are more-or-less comparable to the representative soil properties considered in this study, the proposed method can be applied as a design aid for calculating approximate displacements for any loading on the footing. Moreover, this study presents a methodology by which an empirical method can be developed for any locality, provided that extensive statistical analyses are carried out for obtaining representative soil parameters applicable to the locality.

Figure 8



Load displacement responses predicted by the proposed equation

CONCLUSIONS

The aim of this study was to develop a methodology for obtaining a methodology to formulate a load-displacement equation for square footings embedded in Dhaka soil. Representative parameters of Dhaka soil have been considered from a number of soil investigation reports and, eventually, the load-displacement (P-δ) relationship of square footings has been found to be related by a hyperbolic function, $P = (A\delta/(B+\delta))$; A and B being constants.

The ensuing load-displacement equation traced the finite element predictions well. The resulting load-displacement relationship of square footings may be conveniently used for calculating expected settlements of such footings of a superstructure. Apart from assessing differential settlements, which may be conveniently used as inputs in the analysis of framed structures, footing sizes and depths may be chosen, albeit approximately, using the equation developed, *via* settlement equalization of footings.

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