

COMPRESSIVE-FORCE PATH CONCEPT IN UNIFYING THE DESIGN OF STRUCTURAL CONCRETE

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ABSTRACT

The work presently described aims at introducing a unified approach to the design of reinforced- and prestressed-concrete (RC, PSC) structures, determinate or indeterminate, made from all ranges of concrete strengths. Physical models - based on the compressive-force path (CFP) concept - are presented for the realistic design of RC and PSC beams, shear walls, cantilever beam, coupling beam, continuous beam and fixed-ended portal frames. Experimental verifications are provided to demonstrate that, compliance with the CFP provisions results in a design solution that fulfils the main design objectives for safety (strength and ductility) and economy. The use of the same design equation in all areas of structural concrete design is to be noted. Finally, a unification of design concepts, based on a true understanding of concrete at the material level, may be envisaged.

INTRODUCTION

It can be argued that the design procedures for concrete structures should be based on realistic physical models and not solely on empirical equations, as rational models enable engineers to develop a better understanding of the actual structural behaviour. In this regard, the unsatisfactory nature of the shear design provisions of the present Codes becomes apparent from their collection of complex, restrictive empirical equations. It would appear that it is due to the existence of a general lack of understanding at the material level that the design procedures for structural concrete, put forward by the current Codes, are unnecessarily complicated and do not always yield safe design solutions.

Although new concepts have recently evolved in the general field of structural concrete design, most of these deviate very little from the basis on which present-day design is founded; and, thus, they carry implicit assumptions which, in many cases, are incompatible with the fundamental properties of concrete. The concept of CFP, on the other hand, departs radically from the established design concepts. It was the consideration of the presence of multiaxial stresses in a concrete structure, the recognition of concrete as a brittle material, and the concerns regarding the shortcomings associated with such concepts as the "truss-analogy" and the "aggregate interlock" mechanism, that led to the introduction of the CFP concept [1], which

has proved capable of providing a realistic explanation of the causes of failure of structural concrete.

IMPLEMENTATION IN DESIGN

Physical model for RC and PSC beams

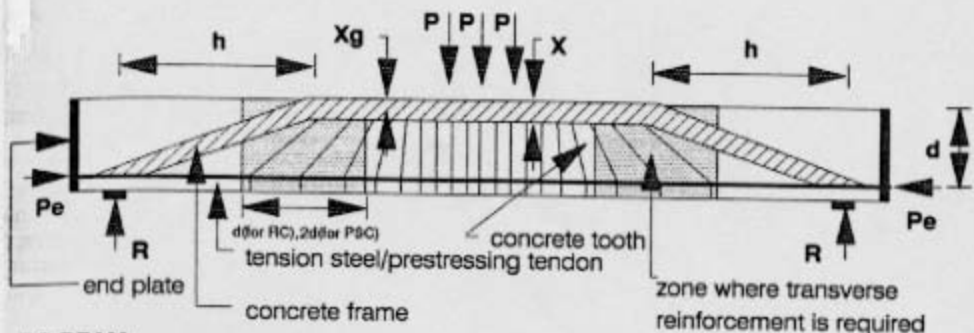
On the basis of the CFP concept, the load-carrying capacity of a structural concrete member is associated with the strength of concrete in the region of the paths along which compressive forces are transmitted to the support. Physical models developed for the design of different structural concrete members are grouped together in Fig. 1. Figure 1a shows the "frame-like" physical model developed for the design of RC [2] and PSC [3,4] beams. The path of the compressive force may be visualized as a "flow" of compressive stresses with varying section perpendicular to the path direction, the compressive force representing the resultant of the stresses at each section. Failure has been shown to be related to the presence of tensile stresses in the region of the path and such stresses may develop due to a number of causes, the main ones being associated with changes in the path direction, the varying intensity of the compressive stress field along the path, bond failure at the level of the tension reinforcement between two consecutive flexural inclined cracks, etc. It can be seen from Fig. 1a that the model comprises a concrete frame with inclined legs, providing a simplified but realistic description of the shape of the CFP, and a number of "teeth" representing concrete cantilevers, which form between consecutive flexural or inclined cracks within the beam web under increasing load.

Physical model for indeterminate members

A cantilever beam subjected to a point loading at its free end can be designed as a simply-supported beam subjected to a point-loading at midspan, since the fixed-end conditions of the cantilever beam are similar to the conditions of the mid-span cross-section of the simply-supported beam. The same model can be used for the design of a structural concrete wall subjected to horizontal loading. Figure 1b shows the application of the model of Fig. 1a to such shear walls and cantilever beams. This method of design has been extended [5] to skeletal structural concrete configurations, with the frame model being used to represent elements between consecutive points of inflection and the regions including such points being modelled as "internal supports". In Fig. 1c, this approach has been adopted in the modelling of a fixed-ended portal frame under the joint action of vertical and lateral loading. Initially, the portal frame has been transformed into a number of determinate structural elements. (The elastic bending moment diagram based on the design load is also shown in this figure.) In this case, however, the design should be complemented so as to allow for the interaction between different determinate portions. The latter interaction is modelled as an "internal support" where the reaction is equal to the shear force that develops in the section through the point of inflection. Figure 1d indicates how a beam with fixed-ends, such as a beam coupling two structural concrete walls, can also be designed in compliance with the CFP method, since it can be divided into two portions between the fixed-end and the section through the point of inflection, each of them being essentially a cantilever beam. Physical model for a two-span continuous RC beam subjected to point loading at the mid-spans is shown in Fig. 1e. It is to be noted that the shape of the physical model for such determinate members, subjected either to point or uniformly distributed loading (udl), can be ascertained by adopting the tied-frame model shown in Fig. 1a.

Criteria of failure

To implement the CFP model in design, it is essential to complement it with a failure criterion. Such a failure criterion cannot be unique, since under the combined action of bending moment and shear force, concrete structures may fail due to a number of causes before flexural capacity is attained, as mentioned earlier. An analytical description of these internal actions



RC BEAM:

$h = 2d$, for point load with $a/d > 2$

$h = a$, for point load with $a/d \leq 2$

$h = 2d$, for udl with $L/d > 8$

$h = L/4$ for $L/d \leq 8$

PSC BEAM:

$h = (d - X_g) * Pe / R$

where,

h = horizontal projection of the inclined portion of the CFP

a = shear span in case of point loading

L = length of the member

Pe = effective prestressing force (=0, for RC member)

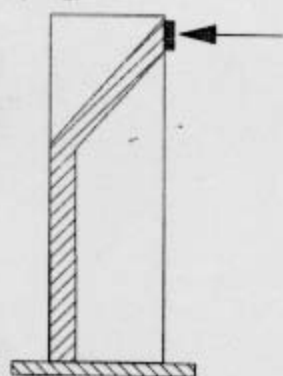
R = reaction at support

P = applied (point or uniformly distributed) load

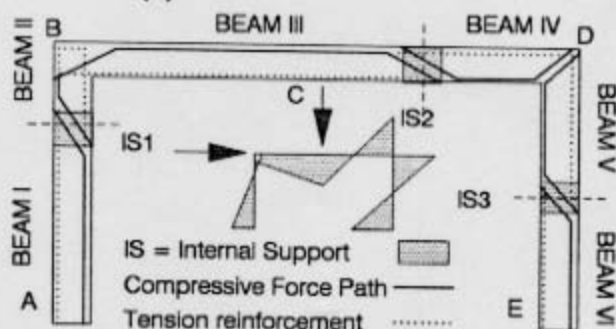
d = effective depth of the member

X_g = centroidal distance of uncracked concrete

(a)



(b)



(c)



(d)

tied-frame model (CFP)



(e)

tension reinforcement

Fig. 1 - Physical models for (a) RC and PSC beams, (b) shear wall and cantilever beam, (c) fixed-ended portal frame, (d) coupling beam, and (e) continuous beam.

causing failure of structural concrete members has already been derived empirically [6], and is shown, as slightly modified elsewhere [2], in the Appendix.

Assessment of transverse reinforcement

If the conditions for failure are fulfilled before flexural failure occurs, one of the design solutions that will allow the beam to attain its flexural capacity involves the provision of transverse reinforcement. It is important to note that such reinforcement is only required, for members with shear span - to - depth ratio (a/d for point loading and M/Vd for udl) ≥ 2 , in the region of the joint of the horizontal and inclined members of the frame, with a nominal amount - as prescribed by current codes - being sufficient in the remaining portions of the member; for members with $1 < a/d < 2$, transverse reinforcement has to be extended up to the support as explained elsewhere [2]. For $a/d \leq 1$, the member has to be designed as a deep beam [2] which is beyond the scope of this paper.

In RC and PSC members, significant internal tensile actions may develop, for equilibrium purposes, within both the regions where the CFP changes direction and, for the case of point loading, the horizontal portion of the path in the region of point loads, to sustain tensile stresses that may develop when bond failure occurs between two consecutive flexural or inclined cracks (bond failure will increase the depth of the right-hand crack, thus causing redistribution as indicated in the Appendix). For simplicity, consider a rectangular cross section. The resulting transverse tensile stresses, with the corresponding stress resultant and the amount of reinforcement required to sustain it, may easily be assessed as described in the Appendix.

The design of the internal support, in case of skeletal structures, involves the provision of transverse reinforcement to counteract the tensile forces ("reactions") acting in that region. Considering the fact that concrete is very weak in tension, only a very minor or no contribution of concrete may be considered in designing such internal supports.

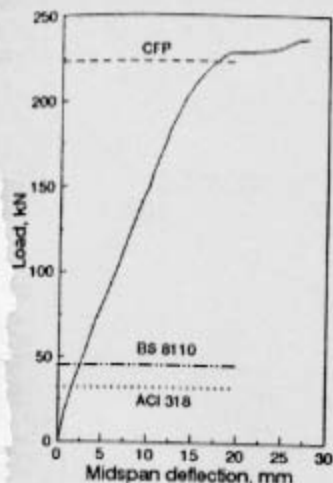
EXPERIMENTAL VERIFICATION

A large amount of physical- and numerical-experiments have been conducted so far in a comprehensive research programme aiming at introducing a unified approach to the design of structural concrete. The present section summarizes some of these findings.

In an attempt to verify the applicability of the CFP method to the design of normal-strength concrete (cube strength ~ 40 MPa) members, Kotsovos and Lefas reported [2] experimental results of a number of 2600 mm long T-beams. On the basis of current design concepts, these beams were deemed incapable of sustaining the design load; yet the beams not only sustained safely the specified design load, but did so with an amount of transverse reinforcement up to 70 percent less than specified by current codes. Figure 2a provides an indication of the disparity between the prediction of CFP and other code provisions for such a four-point loaded T-beam (Beam C in [2]). Clearly, the cause of such predictions should be attributed to the underlying concepts, common to all current shear design methods, rather than the various formulations used by particular methods for the implementation of the concepts in design.

Tests on beams made from high-strength concrete (cube strength ~ 80 MPa) [7], demonstrated that, for a high-strength concrete element to attain its flexural capacity, it does not have to behave like a "truss". Such a member HSB1 underwent a flexural failure at a total load of 227 kN, which was 4.86 (5.5) times greater than the British (American) code prediction; the CFP method predicted the failure load quite accurately. The visible horizontal cracks at the top (compression) flange and spalling of concrete (see Fig. 2b) point to the occurrence of triaxial compressive stress conditions, a feature ignored by the present-day design practice, at the late stages of the loading history of the beam.

The ultimate crack pattern of a repaired RC wall SW 33-R [8], subjected to both horizontal cyclic load and displacement at the top level, is shown in Fig. 2c. The repair work of this



(a)

(b)

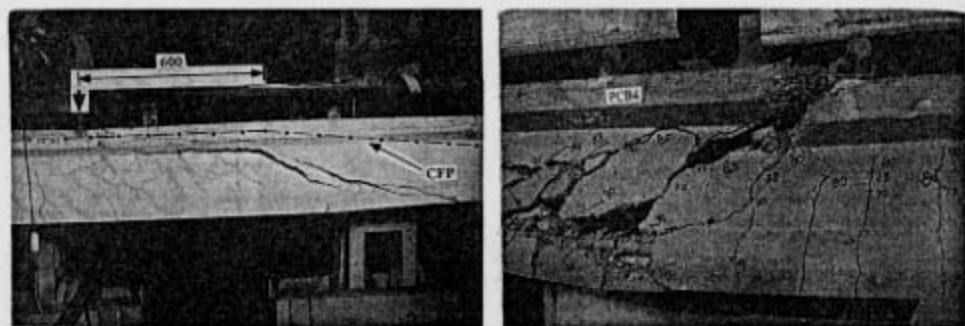
(c)

Fig. 2 - (a) Predicted and experimental load-carrying capacities of normal-strength concrete beam C [2], (b) crack patterns in the zone where failure occurred in high-strength concrete beam HSB1 [7], and (c) final crack patterns in repaired shear wall SW 33-R [8]

specimen involved the replacement of the damaged concrete in the compressive zone only. No special treatment was carried out for the previously yielded vertical bars and the confinement reinforcement of the edge members. It was found that, where repairing only the damaged regions (shown hatched in Fig. 2c) of the compressive zone was sufficient to fully restore wall strength, the additional use of epoxy resins, in similar other specimens, to heal major flexural and inclined web cracks led only to a marginal improvement of the structural characteristics. The observed crack patterns and failure modes of both the original and repaired walls were in compliance with the CFP concept, since they indicated that the wall capacity is associated with the strength of concrete in the region of the compressive zone where the bending moment is maximum and not with the strength of the inclined strut as advocated by the present codes.

Tests on a number of PSC beams [3,4] designed to both conventional and CFP methods have revealed that while beams designed (for flexural failure) to current code provisions may lead to brittle type of failure, comparable girders designed to CFP method attained its full flexural failure load. The ideas on which the CFP method is based were also verified during the course of the experimentation. Very wide (practically see through) cracks (Fig. 3a) were seen in a 6000 mm long PSC beam PCB1 [4] towards the later stages of its loading to failure. Since the so-called "aggregate interlock" mechanism can not be active at a time while such wide cracks are present, the applied load must have been borne by the concrete in the compressive zone since the tension tie (stirrups) of the truss-model was at yield even before the attainment of maximum load and hence this transverse reinforcement could not have provided any additional shear-carrying capacity. The failure crack patterns of a heavily prestressed beam PCB4 (designed to the British Code for flexural failure) [3] is shown in Fig. 3b. Now, bond failure results in an increase in the length of the lever arm (see Fig. A2), which in turn necessitates an enhancement in the average compressive strength of the concrete. Since unlike its CFP counterpart PCB6 [3], beam PCB4 did not have necessary amount of transverse reinforcement in the form of hoops, it was not capable of providing adequate confinement

required for such improvement in the concrete strength. The absence of such reinforcement allowed the sudden proliferation of cracks into the flange. Eventually, the PSC beam PCB4 failed in a quasi-ductile manner, as flange reinforcement usually contributes to the ductility of a reinforced concrete member [2,7].



(a) (b)
Fig. 3 - Crack patterns in the failure zones of PSC beam PCB1 [4], and (b) PCB4 [3]

A pilot study [5] has also shown the beneficial effect of modelling the point of inflection as an "internal support" in indeterminate RC members (like continuous beams and portal frames).

CONCLUSIONS

It appears that design based on the CFP concept leads to safer and economical solution to a wide range of structural concrete members, despite the fact that it differs widely from the current norms of design practice. A reappraisal of the current design concepts, rather than changes in the design equations, should be a prerequisite for any future Code revision. Such a reappraisal is essential not only for improving existing design models, but also for developing new ones. The CFP concept proposes such new models which, eventually may, lead to the unification of the design of structural concrete.

REFERENCES

1. Kotsovos, M.D., "Compressive Force Path Concept : Basis for Reinforced Concrete Ultimate Limit Design," *ACI Structural Journal*, V. 85, No. 1, 1988, pp. 68-75.
2. Kotsovos, M.D. and Lefas, I. D., "Behaviour of reinforced concrete beams designed in compliance with the concept of compressive force path," *ACI Structural Journal*, V. 87, No. 2, 1990, pp. 127-139.
3. Seraj, S. M., Kotsovos, M. D. and Pavlović, M. N., "Compressive-Force Path and Behaviour of Prestressed Concrete Beams," *Journal of Materials and Structures, RILEM*, V. 26 (forthcoming).
4. Seraj, S. M., Kotsovos, M. D. and Pavlović, M. N., "Experimental Study of the Compressive-Force Path Concept in Prestressed Concrete Beams," *Journal of Engineering Structures* (forthcoming).

- Seraj, S. M., Kotsovos, M.D., and Pavlović, M.N., "Application of the Compressive-Force Path Concept in the Design of Reinforced Concrete Indeterminate Structures: A Pilot Study," submitted for publication.
- Bobrowski, J. and Bardhan-Roy, B.K., "Method of Calculating the Ultimate Strength of Reinforced and Prestressed Concrete Beams in Flexure and Shear," *The Structural Engineer*, V. 47, 1969, pp. 197-209.
- Seraj, S. M., Kotsovos, M.D., and Pavlović, M.N., "Behaviour of high-strength mix reinforced concrete beams," Submitted for publication.
- Lefas, I. D. and Kotsovos, M.D., "Strength and Deformation Characteristics of Reinforced Concrete Walls under Load Reversals," *ACI Structural Journal*, V. 87, No. 6, 1990, pp. 716-726.

APPENDIX

Failure criterion

$$M_c = 0.875sd(0.342b_1 + 0.3(M_f/d^2)(z/s)^{1/2})(16.66/\rho_w f_y)^{1/4} \quad (1)$$

where, M_c is moment the moment at failure load (Nmm), b_1 is the effective width (see Fig. 4a), M_f is the flexural capacity (Nmm), ρ_w is the tension steel ratio, s is the distance from support of cross-section at which M_c is calculated (mm) (equal to shear span for point loading, and $2d$ (in RC members) or h (in PSC members) for uniformly distributed loading), z is the lever arm and f_y is the characteristic strength of the tension steel.

Eq. 1 may be used in RC/PSC member design as follows:

- Select cross-section s ;
- Find moment M_a at cross section s due to applied loading;
- Design cross-section s to sustain a given M_f .
- Determine M_c from Eq. 1 (If $M_c > M_a$ only nominal stirrups would be needed. Otherwise, if $M_c < M_a$, either increase area of tension steel (thus increasing M_c to a level greater than or equal to M_a), or increase the cross-section; the alternative is to provide transverse reinforcement in accordance with the requirements described in the Appendix).

Assessment of transverse reinforcement

(a) Region where CFP changes direction

Excess tensile force $T_{sv} = V = V_a - V_c (= M_c / s)$

Transverse reinforcement over a length d to sustain T_{sv} will be, $A_{sv} = T_{sv} / f_{yv}$

(b) Horizontal portion of path (see Fig. 4b)

Using information in Fig. 4b, the following steps are used:

1. $\Delta z = V * X / (2T)$; $V = V_a - V_c$

2. $x' = 2(d - z - \Delta z) > 0$; if $x' < 0$, increase cross-section

3. Assess nominal triaxial compressive stress $\sigma_c' = C / (bx')$

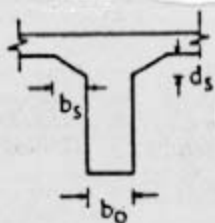
4. Assuming $0.8 f_{cy1}$ describes uniaxial condition ($\sigma_c = 0.8 f_{cy1}$), assess confining pressure

σ_{conf} required for σ_c to increase to σ_c' , from expression $\sigma_{conf} = (\sigma_c' - 0.8 f_{cy1}) / 5$

5. Assume transverse tensile stress $\sigma_t = -\sigma_{conf}$ (compression positive)

6. Tensile force over length δ will be $T_{sv} = \sigma_t * b * \delta$

7. Amount of transverse reinforcement over length δ to sustain tension T_{sv} , $A_{sv} = T_{sv} / f_{yv}$



$$b_1 = b_o + 2d_s \text{ or } b_o + 2b_s$$

(whichever is smaller)

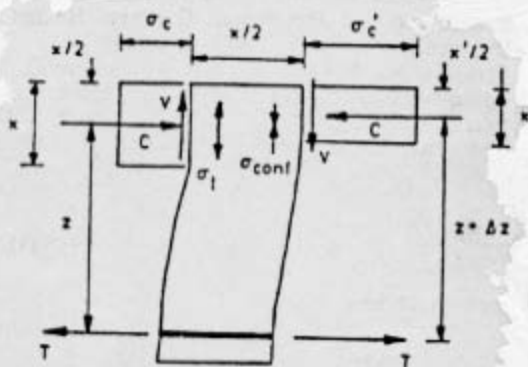


Fig. 4 - (a) Definition of b_1 , and (b) assessment of excess tension due to bond failure